A Polynomial Algorithm for Subisomorphism of Open Plane Graphs

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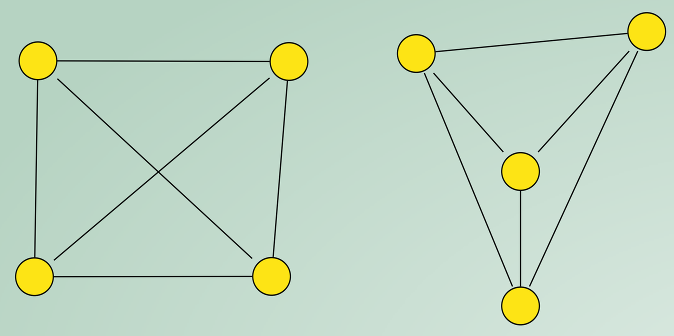
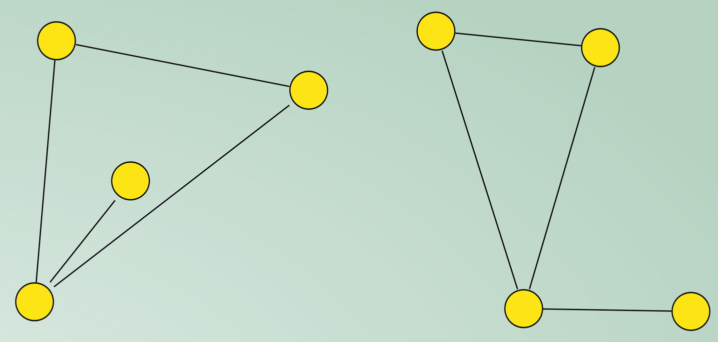
The issue

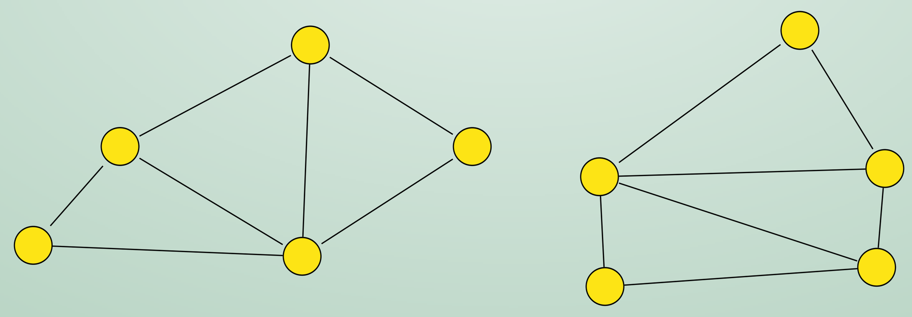
- Is it the same object? (isomorphism)
- Is it a part of the same object? (subisomorphism)
- Can we forget about all the background? (open graph)



These two graphs are isomorphic.



These two plane graphs are not (plane graph-)isomorphic.



These two plane graphs are (plane graph-)isomorphic.

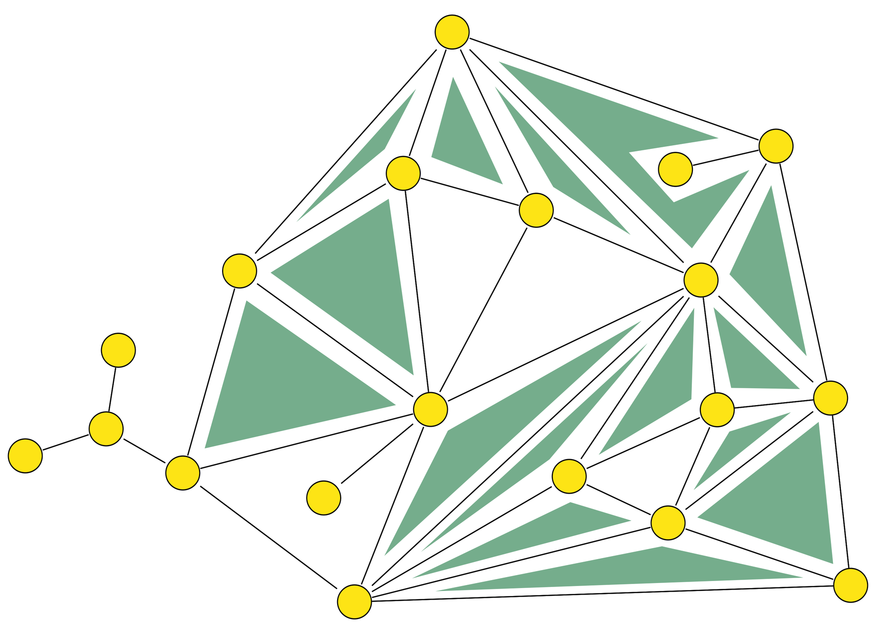
The correct (sub)isomorphism

Normalising a graph G:

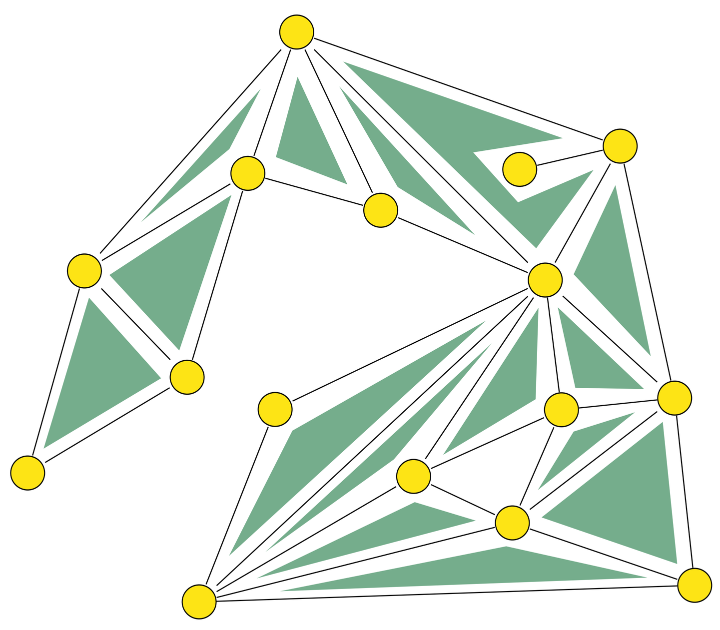
- eliminate all invisible nodes and edges (bridges)
- duplicate all vertices invisible from two sides (hinges)

The resulting graph is called irreducible and denoted by $N(G)$.

Two irreducible graphs $G=(X,E,F,V,e)$ and $G'=(X',E',F',V',e')$ are isomorphic if there is a bijection: $X \rightarrow X'$ which respects the vertices, faces, visible faces and the external face. Two graphs G and G' are equivalent if $N(G) \equiv N(G')$.



before normalisation

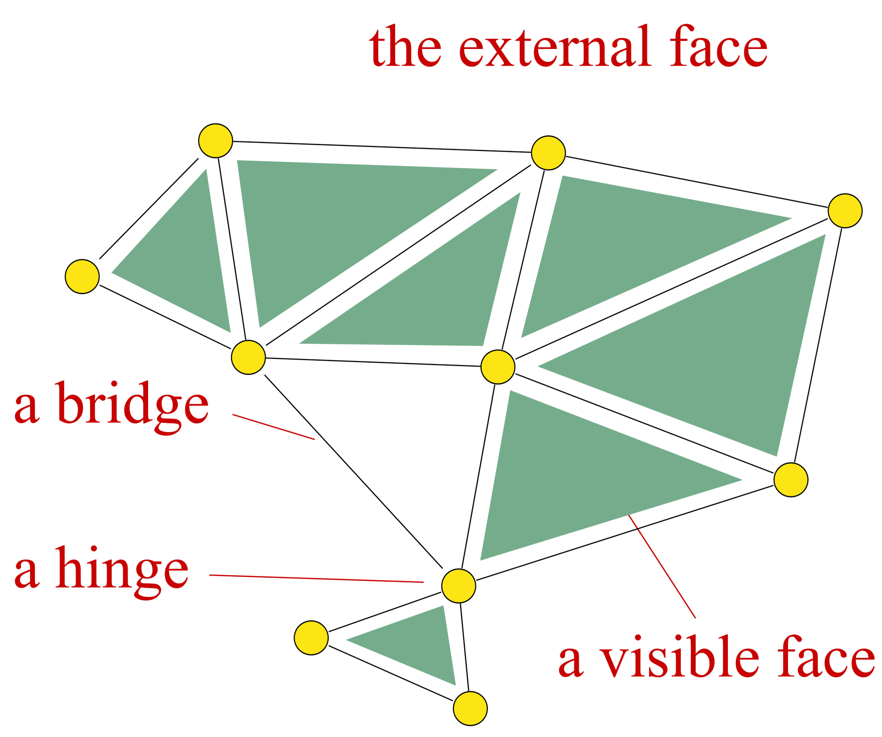


the irreducible form

$G=(X,E,F,V,e)$ is a subgraph of $G'=(X',E',F',V',e')$ if there exists a graph $G''=(X',E',F',V'',e')$ with $V'' \subseteq V'$ and $G \equiv N(G'')$.

There exists a subisomorphism between G and G' if G is a subgraph of G'.

The objects



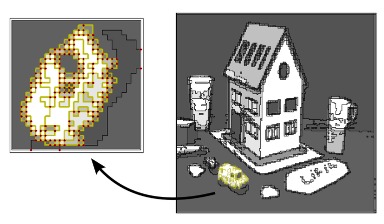
An open plane graph is composed of:

- a set of vertices X
- a set of edges $E \subset X^2$
- a set of faces $F \subset X^*$
- a set of visible faces $V \subset F$
- an external face $e \in F$

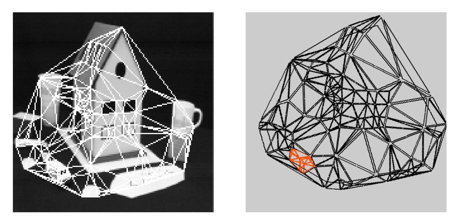
An open plane graph is face-connected if between any two faces, there is a path of faces (in such a path, two consecutive faces are separated by one or more edges).

Experiments

Finding patterns in thumbnail images.



Combinatorial map obtained by segmentation



Delaunay triangulation of the interest points

size graph \ subgraph	10% nodes vf2	10% nodes map	33% nodes vf2	33% nodes map	50% nodes vf2	50% nodes map
5000	0.04	0.10	0.7	0.02	10.4	0.10
10000	2.54	0.07	7.31	0.06	12.7	0.06
50000	156.5	0.31	>3600	0.31	>3600	0.31

Comparison of scale-up properties of subgraph and submap isomorphism algorithms. Time in seconds.

A full description of the experiments can be found in the paper from [GbR09].

Theorems

- Plane isomorphism ∈ P for irreducible connected open plane graphs.
- Equivalence ∈ P for face-connected open plane graphs.
- Subisomorphism ∈ P for irreducible connected open plane graphs.

Proof

In the above cases, the graphs can be transformed into combinatorial maps and we can use techniques from [GbR09].

References

[GbR09] G. Damiand, C. de la Higuera, J.-C. Janodet, É. Samuel and C. Solnon. A Polynomial Algorithm for Submap Isomorphism: Application to Searching Patterns in Images. *GbRPR 09*, *LNCS* 5534, pp.102-112 (2009)

[Cor04] L. P. Cordella , P. Foggia , C. Sansone , M. Vento. A (Sub)Graph Isomorphism Algorithm for Matching Large Graphs. *IEEE Trans. Pattern Anal. Mach. Intell.* 26(10): 1367-1372 (2004)