Non-stationary Multicomponent Signals Analysis using the Synchosqueezing Transform

Dominique Fourer

IBISC - Team SIAM

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Plan

A short bio...

MCF61 IUT d'Evry, dept TJC, (qualif. CNU 26, 27, 61).

Postes occupés

- 2016-2018 : postdoc à l'IRCAM (H2020 ABC-DJ)
- 2015-2016 : postdoc à l'IREENA (Saint-Nazaire) + LJK (Grenoble) projet ANR ASTRES (1 an + 6 mois mi-temps)
- 2014-2015 : postdoc au LaBRI, projet ANR DIADEMS
- 2009-2013 : doctorant+ATER au LaBRI, université de Bordeaux 1
- avant 2009 : ingénieur R&D : SC2X S.A.S (stage Bordeaux), Sisenior SPRL (développeur freelance, Bruxelles, BE),

Formation

- Doctorat informatique : "Approche informée pour l'analyse du son et de la musique" (direc. S. Marchand). dec. 2013. (EDMI, univ. Bordeaux 1)
- Master 2 mathématiques appliquées : "Traitement harmonique et contrôle du signal" (univ. Bordeaux 1)
- Master 2 informatique : "Image et son, multimédia" (univ. Bordeaux 1)

Expériences et compétences

Thèmes de recherche

- Traitement du signal : analyse temps-fréquence, analyse spectrale, codage de sources, problèmes inverses, séparation de sources
- Extraction d'informations musicales (MIR) : estimation F_0 multiple, reconnaissance du timbre instrumental, apprentissage machine
- Traitement de la parole : caractérisation et modélisation des attitudes socio-culturelles (analyse statistique et apprentissage machine)

Projets de recherche

- H2020, ABC-DJ (IRCAM *et al.* , 2016-2018)
- ANR ASTRES (ENS LYON, LJK, IREENA, 2013-2017)
- ANR DIADEMS (IRIT, LaBRI, LIMSI, LAM, CREM, 2012-2015)
- PEPS/IDEX, univ. Bordeaux (2013-2014)
- ANR DReAM (LaBRI, Lab-STICC, TSI, GIPSA-lab, 2009-2014)

Publications

Plus de 30 articles publiés dans des revues (5+) et conférences internationales (25+).

Tâches collectives et vie associative

Activités de relecture

- Journaux internationaux : IEEE TSP, SPL, JASA. Math. Geosciences
- Conférences internationales : ICASSP, ISMIR, EUSIPCO, DAFx, ...

Tâches collectives

- Membre IEEE, de l'AFCP (Association Francophone de la Communication Parlée),
- Membre du LaBRUIT (jusqu'en 2013),
- Développeur et webmaster du site de l'EDMI (école doctorale maths-info) de Bordeaux,
- Développeur et webmaster du site du projet ANR DREAM.

Divers

- Musicien de jazz (piano), formation conservatoire (CRR Bordeaux)
- Associé, YES WE CARE SPRL (www.myspecialist.be), BE 0845.536.330 - Siège social : Avenue Grandchamp, 93 - 1150 Bruxelles, Belgique.

Purpose of this presentation

Goals

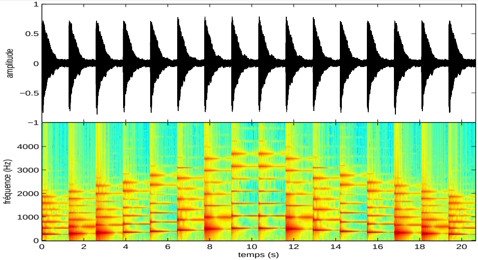
- Meaningful **information extraction** from observation (signal or image) .
- Computing efficient representations designed for **classification/detection/prediction** problems.
- **Separation/clustering** of elementary signal components (atoms).

Investigated approaches

- **Time-frequency**/Time-scale reassignment-based transformations (i.e. synchosqueezing)
- ~~Data-driven empirical signal decomposition (e.g. EMD, SSA)~~ $\Rightarrow$ in a future presentation.

$\Rightarrow$ **r**ationales and open research directions

Why time-frequency analysis (TFA) ?



Waveform and spectrogram of a piano playing the C major scale.

Motivations : non-stationary multicomponent signals analysis/modeling

TFA can :

- help to disentangle/segregate elementary components (*i.e.* sinusoids, noise, transients, etc.)
- lead to sparse representation (useful for data compression)
- lead to efficient and physically meaningful representations to be combined with a machine learning approach
- provide intuitive and readable representations similar to a music score (related to "instantaneous frequency" [Ville, 48])

Principle and limitations

Linear transform of a signal x using a time-frequency decomposition

$$\Phi_x(\gamma) = \int_{-\infty}^{+\infty} x(t)\phi_\gamma^*(t)\, dt \qquad (1)$$

with $x, \phi_\gamma \in L^2(\mathbb{R})$ et $||\phi_\gamma||^2 = \int_{\mathbb{R}} |\phi_\gamma(t)|^2\, dt = 1$

Heisenberg-Gabor uncertainty principle (1927)

$$\boxed{\sigma_t \sigma_\omega \geq \frac{1}{2}} \qquad (2)$$

with :

$$\sigma_t^2 = \int_{-\infty}^{+\infty} (t-\bar{t})^2 |\phi_\gamma(t)|^2\, dt \text{ et } \bar{t} = \int_{\mathbb{R}} t|\phi_\gamma(t)|^2\, dt$$

$$\sigma_\omega^2 = \frac{1}{2\pi}\int_{-\infty}^{+\infty} (\omega-\bar{\omega})^2 |\hat{\phi}_\gamma(\omega)|^2\, d\omega \text{ et } \bar{\omega} = \frac{1}{2\pi}\int_{\mathbb{R}} \omega|\hat{\phi}_\gamma(\omega)|^2\, d\omega$$

with $\hat{\phi}(\omega) = \int_{\mathbb{R}} \phi_\gamma(t)\, \mathbf{e}^{-j\omega t}\, dt$, the Fourier transform of $\phi_\gamma(t)$.

Definitions 1 : Short Time Fourier Transform (STFT)

x being the analyzed signal and h differentiable analysis window h.

Analysis

$$F_x^h(t,\omega) = \int_{\mathbb{R}} x(u)h(t-u)^* \, \mathbf{e}^{-j\omega u} \, \mathrm{d}u, \tag{3}$$

- A time-frequency representation is provided by $|F_x^h(t,\omega)|^2$ called spectrogram.
- When $h(t) = \frac{1}{2\pi\sqrt{T}} \, \mathbf{e}^{-\frac{t^2}{2T^2}}$, $F_x^h(t,\omega)$ is called the gabor transform.

Reconstruction formulas

$$\hat{x}(t) = \iint_{\mathbb{R}^2} F_x^h(\tau,\omega) \, \mathbf{e}^{j\omega\tau} \, \mathrm{d}\tau \frac{\mathrm{d}\omega}{2\pi} \tag{4}$$

and its simplified reconstruction formula (when $h(t_0) \neq 0$, $\forall t_0 \geq 0$) :

$$x(t-t_0) = \frac{1}{h(t_0)} \int_{-\infty}^{+\infty} F_x^h(t,\omega) \, \mathbf{e}^{j\omega(t-t_0)} \, \frac{\mathrm{d}\omega}{2\pi}. \tag{5}$$

Definitions 2 : Continuous Wavelet Transform

Continuous Wavelet Transform

For an admissible (orthogonal) mother wavelet function Ψ (*i.e.* $C_\Psi = \int_{\mathbb{R}} \frac{|\hat{\Psi}(\omega)|^2}{|\omega|} d\omega < +\infty$).

$$W_x^\Psi(t,s) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{+\infty} x(\tau)\Psi\left(\frac{\tau - t}{s}\right)^* \mathrm{d}\tau \quad (6)$$

$s = \frac{\omega_0}{\omega}$ (s being a scale and ω_0 the center frequency of Ψ). The scalogram can be computed by $|W_x(t,s)|^2$ or $|CW_x(t,\omega)|^2$.

The Morlet wavelet is given by $\Psi(t) = \frac{\pi^{-1/4}}{\sqrt{T}} \mathbf{e}^{\frac{-t^2}{2T^2}} \mathbf{e}^{j\omega_0 t}$,

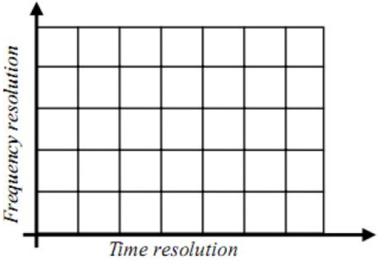
Reconstruction formula

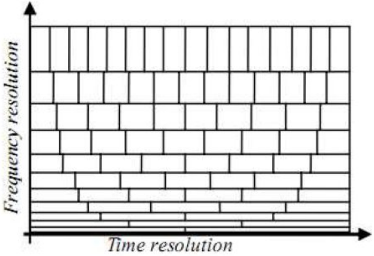
The synthesis (Morlet) formula is given by :

$$x(t) = \frac{1}{C_\psi} \int_{\mathbb{R}} W_x^\Psi(t,s)|s|^{-3/2} ds \text{ , with } C_\psi = \int_{\mathbb{R}} F_\Psi(\omega)^* \frac{\mathrm{d}\omega}{\omega} \quad (7)$$

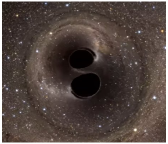
where $F_\Psi(\omega)$ denotes the Fourier transform of $\Psi(t)$.

STFT vs CWT

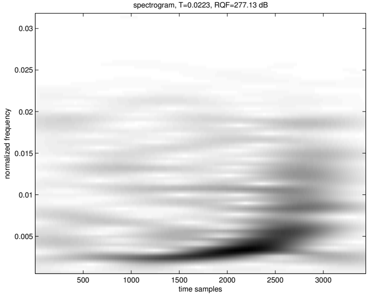

Frequency resolution
Time resolution


Frequency resolution
Time resolution

Example : Analysis of the Livingston GW140915 signal

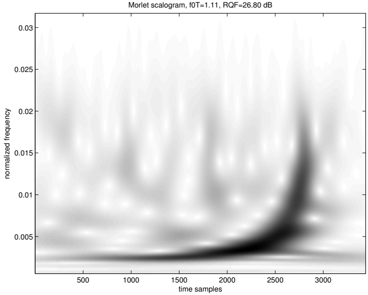


STFT



$|F_x^h(t,\omega)|^2$

CWT



$|W_x(t,\omega)|^2$

D. Fourer, J. Harmouche, J. Schmitt, T. Oberlin, S. Meignen, F. Auger and P. Flandrin. The ASTRES Toolbox for Mode Extraction of Non-Stationary Multicomponent Signals. Proc. EUSIPCO 2017, Aug. 2017. Kos Island, Greece.

The reassignment method [Kodera *et al.* 1978] [Auger & Flandrin 1995]

TF reassignment improves the energy concentration (readability) of any bilinear distribution by reassigning its energy to new locations closer to real signal support.

Considering a time-frequency representation (TFR) of a signal x expressed in terms of the Wigner-Ville distribution as :

$$\mathrm{TFR}_x(t,\omega) = \iint_{\mathbb{R}^2} \mathrm{WV}_x(\tau,\Omega)\Phi(t-\tau,\omega-\Omega)d\tau d\Omega$$

Method description

- Computation of the reassignment operators :

$$\hat{t}(t,\omega) = \frac{\int_{\mathbb{R}^2} \tau \mathrm{WV}_x(\tau,\Omega)\Phi(t-\tau,\omega-\Omega)d\tau d\Omega}{\int_{\mathbb{R}^2} \mathrm{WV}_x(\tau,\Omega)\Phi(t-\tau,\omega-\Omega)d\tau d\Omega} \quad (8)$$

$$\hat{\omega}(t,\omega) = \frac{\int_{\mathbb{R}^2} \Omega \mathrm{WV}_x(\tau,\Omega)\Phi(t-\tau,\omega-\Omega)d\tau d\Omega}{\int_{\mathbb{R}^2} \mathrm{WV}_x(\tau,\Omega)\Phi(t-\tau,\omega-\Omega)d\tau d\Omega}. \quad (9)$$

- Computation of the reassigned time-frequency representation :

$$\mathrm{RTFR}_x(t,\omega) = \int_{\mathbb{R}^2} \mathrm{TFR}_x(\tau,\Omega)\delta(t-\hat{t}(\tau,\Omega))\delta(\omega-\hat{\omega}(\tau,\Omega))\, d\tau d\Omega \quad (10)$$

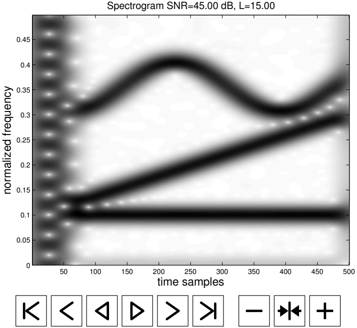
Example : the reassigned spectrogram

$F_x^h(t,\omega) = \int_{\mathbb{R}} x(\tau) h(\tau - t)^* \, e^{-j\omega\tau} \, d\tau$ being the STFT of a signal x using analysis window h.

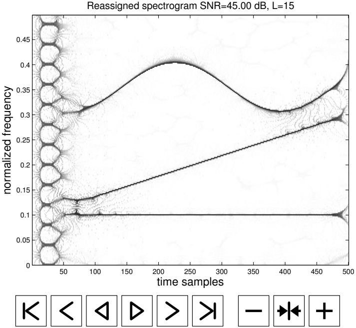
$$\hat{t}(t,\omega) = -\frac{\partial \Phi_x^h}{\partial \omega}(t,\omega) = t + \mathrm{Re}\left(\frac{F_x^{Th}(t,\omega)}{F_x^h(t,\omega)}\right) \quad \text{, with } Th(t) = t\, h(t) \tag{11}$$

$$\hat{\omega}(t,\omega) = \omega + \frac{\partial \Phi_x^h}{\partial t}(t,\omega) = \omega + \mathrm{Im}\left(\frac{F_x^{Dh}(t,\omega)}{F_x^h(t,\omega)}\right) \quad \text{, with } Dh(t) = \frac{dh}{dt}(t) \tag{12}$$

$$\mathrm{R}_x(t,\omega) = \iint_{\mathbb{R}^2} \left| F_x^h(\tau,\Omega) \right|^2 \delta(t - \hat{t}(\tau,\Omega))\delta(\omega - \hat{\omega}(\tau,\Omega)) \, d\tau \frac{d\Omega}{2\pi} \tag{13}$$



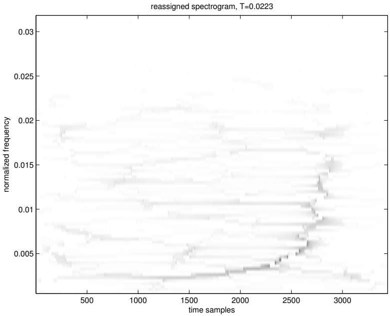
(a) $|F_x^h(t,\omega)|^2$



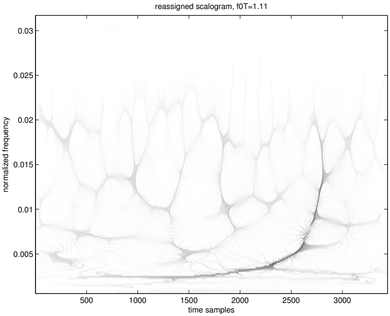
(b) $|R_x^h(t,\omega)|^2$

Example : TF Reassignment of the Livingston GW140915 signal

STFT



CWT



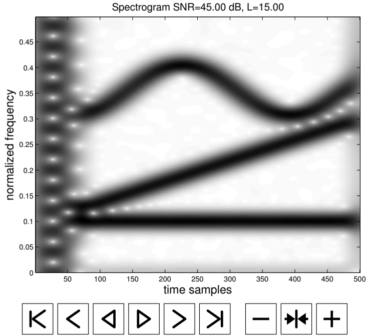
Synchrosqueezing

Can be viewed as a particular reassignment method which allows to compute sharpen and reversible TFRs [Daubechies 1996, 2011] [Thakur 2011].

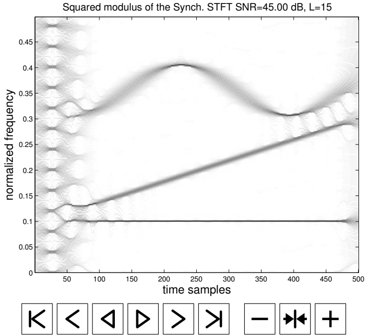
Computation of the synchrosqueezed STFT and of its signal reconstruction formula :

$$\mathbf{S}_{\boldsymbol{x}}(\boldsymbol{t}, \omega) = \frac{1}{\boldsymbol{h}(0)} \int_{\mathbb{R}} \boldsymbol{F}_{\boldsymbol{x}}^{\boldsymbol{h}}(\boldsymbol{t}, \Omega)\delta(\omega - \hat{\omega}(\boldsymbol{t}, \Omega)) \frac{\boldsymbol{d}\Omega}{2\pi} \quad (14)$$

$$\hat{\boldsymbol{x}}(\boldsymbol{t}) = \int_{\mathrm{supp}_{\Omega}(\boldsymbol{x})} \mathbf{S}_{\boldsymbol{x}}(\boldsymbol{t}, \Omega)\boldsymbol{d}\Omega \quad (15)$$



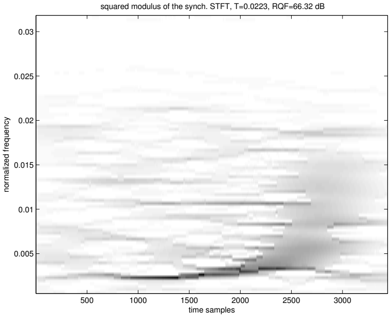
(c) $|F_{x}^{h}(t, \omega)|^2$



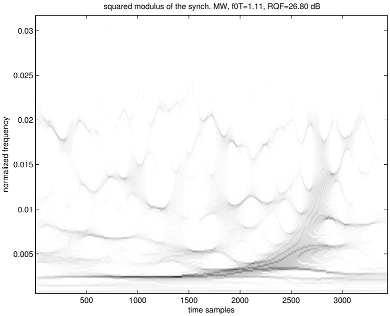
(d) $|S_{x}(t, \omega)|^2$

Example : Synchrosqueezing of the Livingston GW140915 signal

STFT



CWT



Second-order synchrosqueezing [Oberlin et al. 2014] [Behera et al. 2016]

Goal : to improve TFRs of strongly modulated chirps signal.

New considered signal model

$$x(t) = a(t)\,\mathbf{e}^{j\Phi(t)}, \text{ with } \Phi(t) = \varphi_x + \omega_x t + \alpha_x \frac{t^2}{2}, \tag{16}$$

Enhanced IF estimation

Local modulation estimation :

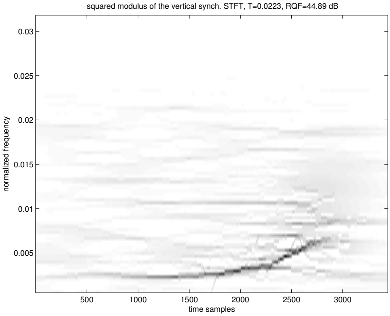
$$\hat{\alpha}_x(t,\omega) = \frac{\frac{\partial \hat{\omega}(t,\omega)}{\partial t}}{\frac{\partial \hat{t}(t,\omega)}{\partial t}} = \mathrm{Im}\left(\frac{F_x^{\mathcal{D}^2 h}(t,\omega) F_x^h(t,\omega) - F_x^{\mathcal{D}h}(t,\omega) F_x^{\mathcal{D}h}(t,\omega)}{F_x^{\mathcal{D}h}(t,\omega) F_x^{\mathcal{T}h}(t,\omega) - F_x^{\mathcal{T}\mathcal{D}h}(t,\omega) F_x^h(t,\omega)} \right) \tag{17}$$

Enhanced IF estimator :

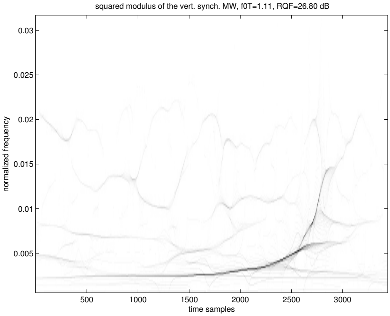
$$\hat{\omega}^{(2)}(t,\omega) = \begin{cases} \hat{\omega}(t,\omega) + \hat{\alpha}_x(t,\omega)(t - \hat{t}(t,\omega)) & \text{if } |\hat{\alpha}_x(t,\omega)| < \infty \\ \hat{\omega}(t,\omega) & \text{otherwise,} \end{cases} \tag{18}$$

Example : Vertical synchrosqueezing of the Livingston GW140915 signal

STFT



CWT



Mode extraction [Brevdo, 2011]

Goal : Automatic extraction of signal components.

Considering the synchrosqueezing transform of a signal $S_x(t,\omega)$, this technique finds the best frequency curve $\Omega(t)$ which maximizes the energy with a smooth constraint through a total variation term penalization obtained by :

$$\hat{\Omega} = \operatorname*{argmax}_{\Omega} \int_{\mathbb{R}} |S_x(t,\Omega(t))|^2 \, dt - \lambda \int_{\mathbb{R}} \left| \frac{d\Omega}{dt}(t) \right|^2 dt, \tag{19}$$

where λ controls the importance of the smoothness of Ω.

When the ridges of several components have to be estimated, this method can be iterated after subtracting the energy located at the previously estimated ridge.

Audio source separation using AM/FM local estimation [Fourer et al. 2018]

Principle : We use TF local modulations to segregate independently evolving sources.

Assumptions : Sources are disjoints in the TF plane (each TF point is assigned to one source).

Mixture model

$$x(t) = \sum_{c=1}^{C} s_c(t) = \sum_{c=1}^{C} \left(\sum_{i_c \in \mathcal{I}_c} \mathrm{e}^{\lambda_{i_c}(t) + j\phi_{i_c}(t)} \right) \tag{20}$$

$$\text{with} \quad \lambda_x(t) = l_x + \mu_x t + \nu_x \frac{t^2}{2} \tag{21}$$

$$\text{and} \quad \phi_x(t) = \varphi_x + \omega_x t + \alpha_x \frac{t^2}{2} \tag{22}$$

Coherent local modulation used for source separation

$$\mathrm{CFM}_x(t,\omega) = \frac{\hat{\alpha}_x(t,\omega)}{\hat{\dot{\phi}}_x(t,\omega)}, \quad \mathrm{CAM}_x(t,\omega) = \frac{\hat{\dot{\lambda}}_x(t,\omega)}{\hat{l}_x(t,\omega)}. \tag{23}$$

Results on real-world music signals (in terms of BssEval)

Table : Comparison of the voice/guitar separation results for the music piece AlexanderRoss VelvetCurtain [MedleyDB].

(a) New proposed estimator ($\omega 2$)

Method	RQF (dB)	SIR (dB)	SDR (dB)	SAR (dB)
Oracle	8.70/9.66	19.02/23.21	8.08/9.24	8.50/9.44
CFM + k-means	**5.88/6.02**	9.77/11.14	**4.58/4.84**	**6.58**/6.32
CAM + k-means	2.47/2.62	2.57/11.70	0.23/1.49	0.84/**9.96**
CFM/CAM + k-means	2.99/3.16	3.65/**12.16**	0.63/1.93	1.20/8.36

(b) baseline modulation estimator (SM12)

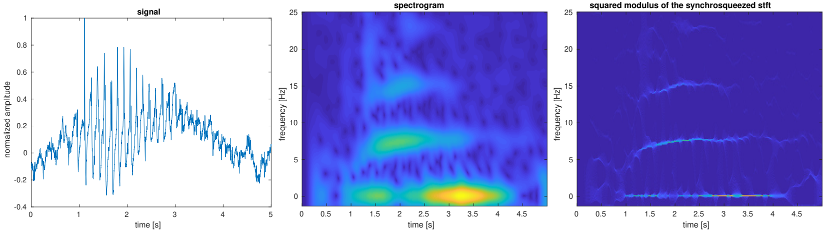
Method	RQF (dB)	SIR (dB)	SDR (dB)	SAR (dB)
Oracle	9.05/9.60	19.14/23.58	8.50/9.18	8.94/9.36
CFM + k-means	5.43/5.38	8.57/8.63	3.97/3.92	6.38/6.27
CAM + k-means	2.48/2.57	11.19/2.53	-0.14/1.33	0.50/9.41
CFM/CAM + k-means	2.38/2.47	**11.53**/2.46	-0.45/1.27	0.12/9.44

Audio examples at : `http://www.fourer.fr/publi/spl18`

Real-time HVS detection from EEG signals

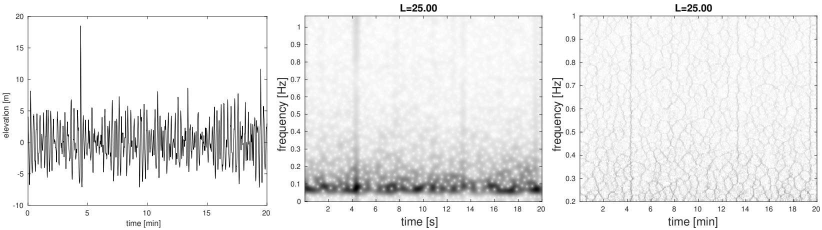
[Souriau, Fourer et al, 2019] (submitted at GRETSI 2019)

Principle : Combining recursive synchrosqueezing [Fourer et al. 2016] with a stochastic detection method.



Draupner wave analysis using time-reassigned synchrosqueezing

Principle : Introduce a new second-order time-reassigned synchrosqueezing method [Fourer, 2019] (submitted to GRETSI 19 and EUSIPCO 19)



The ANR PRC ASCETE project

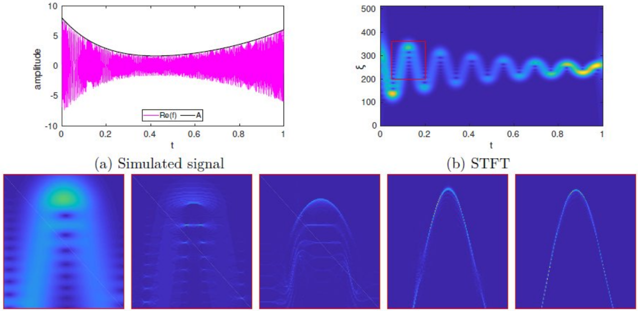
SUPPORTED BY ANR

(with : LJK, ENS LYON, TelecomParisTech, LRI)

Goal : Combining deterministic and stochastic approaches for new signal processing methods

- Combining TF tools with machine learning (e.g. deep neural networks)
- Addressing difficults cases for mode recovery (e.g. overlapping components, noisy signals, etc.)
- Combining synchrosqueezing with Non-negative Matrix Factorization (NMF)
- Generalization to high dimension signals (images, tensors, etc.)
- New practical applications (perception, biomedicine, astronomy, etc.)

Improving the IF and chirp-rate estimation



(a) Simulated signal (b) STFT

(c) STFT zoom (d) FSST zoom (e) FSST2 zoom (f) FSST3 zoom (g) FSST4 zoom

- Investigates new specific signal models (e.g. hyperbolic chirps, modulated impulses, etc.) [Pham et al. 2017] [Behera et al. 2016]
- Deals with overlapping components
- Considers stochastic signal models
- Further investigate signal and its transform properties (marginals, derivatives, reconstruction conditions, etc.) [Fourer et al 2017] [Fourer et al 2018]

Synchrosqueezing/Reassignment of new signal transforms

- Fully adaptive transform (Chirplet transform [Czarnecki,Fourer et al. 2018])
- High-order synchrosqueezing of the S-transform [Fourer, Auger, 2015],[Fourer, Auger, 2017]
- Discrete-time transforms (e.g. CQT, recursive CWT, etc.) [Fenet, Badeau, Richard 2017] [Fourer, Auger 2017]
- Investigate other transforms (with applications) : MDCT (data compression), scattering transform, Mellin (pattern recognition)

Future works perspectives

Proposals and open discussions

- Investigating deep neural networks (in particular CNNs)
- Investigating new applications : image processing, biology, signal processing on graphs.
- Reinforcing and reinterpreting methods with theory (ML and optimization).

Perspectives

- Collaboration with IRCAM (several works in progress related to neural networks).
- Collaboration with IREENA (several works in progress related to energy : NILM and smart grids).
- Collaboration with LaBRI (several works in progress related to perception : audio, speech, image, video processing).