Statistical 3D Shape Analysis Using Square-Root Normal Fields

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Joint work with:

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What is shape

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Shape

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This article is about describing the shape of an object e.g. shapes like a triangle. For common shapes, see list of geometric shapes. For other uses, see Shape (disambiguation).

A **shape** is the form of an object or its external boundary, outline, or external surface, as opposed to other properties such as color, texture, or material composition.

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A **shape** is the form of an object or its external boundary, outline, or external surface, as opposed to other properties such as color, texture, or material composition.

Mathematician and statistician David George Kendall writes:[2]

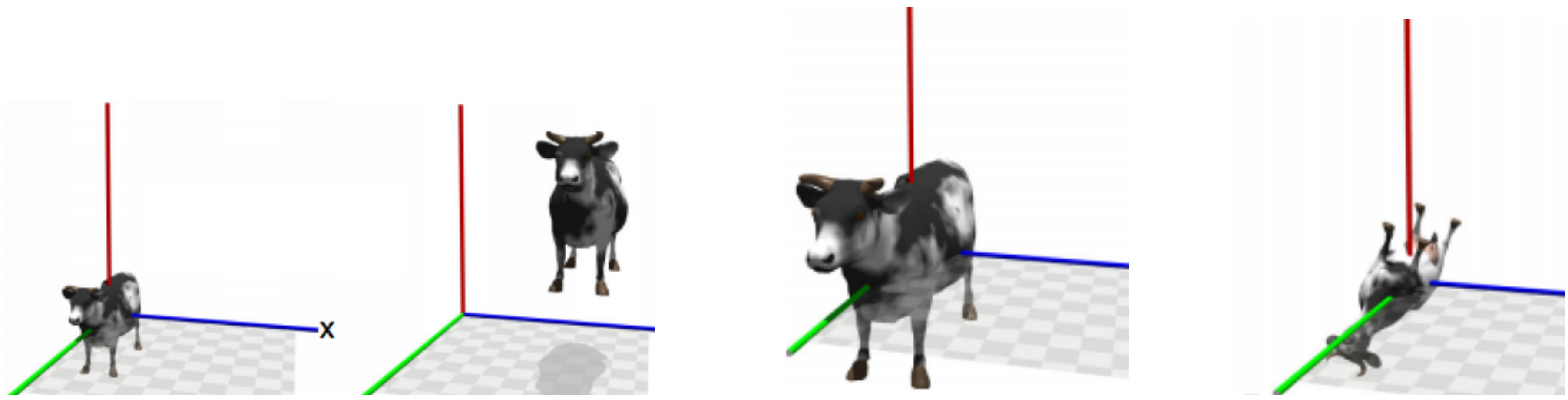
> In this paper 'shape' is used in the vulgar sense, and means what one would normally expect it to mean. [...] We here define 'shape' informally as 'all the geometrical information that remains when location, scale[3] and rotational effects are filtered out from an object.'

What is shape

- Shape is not affected by some shape-preserving transformations

Shape is what is left when differences, which can be attributed to translations, scale, and rotations have been filtered out

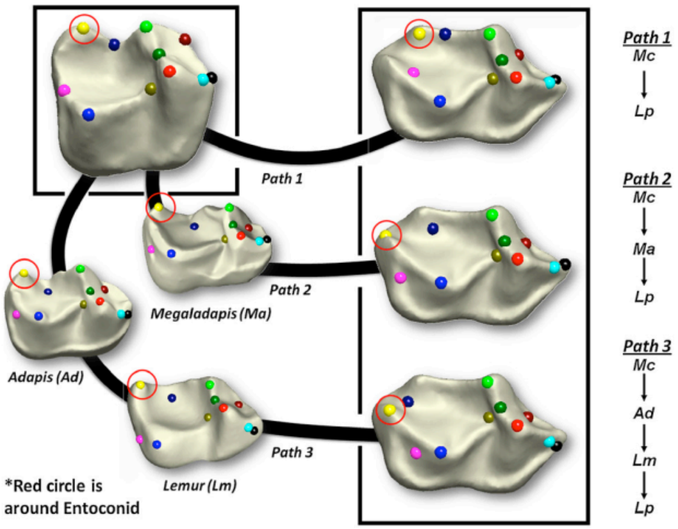
David G. Kendall (1984) – first to introduce statistics into shape analysis



Original Translate Scale Rotate

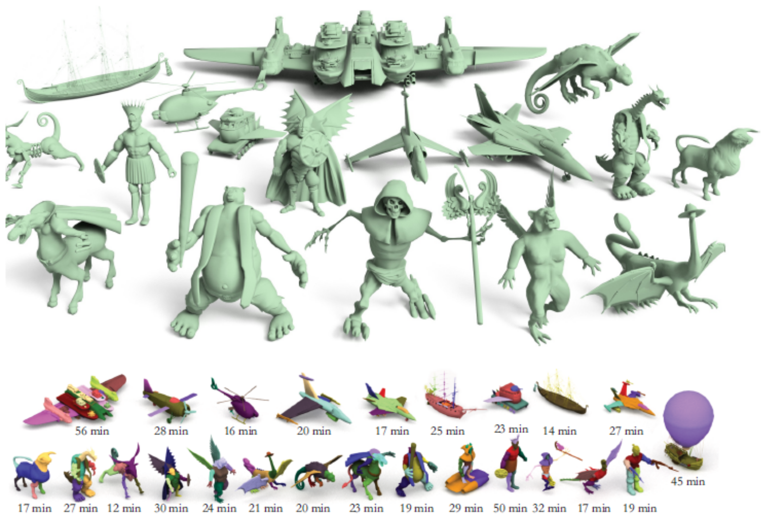
Why shape is important ?

Biomedical



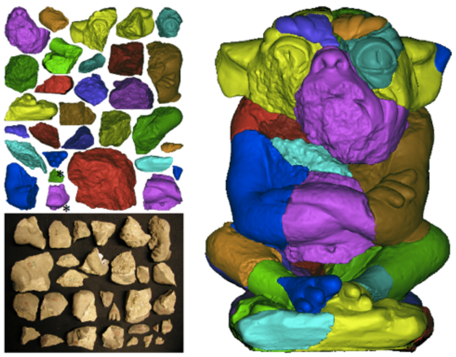
[Boyer et al. 11]

3D modeling



[Chaudhuri et al. 11]

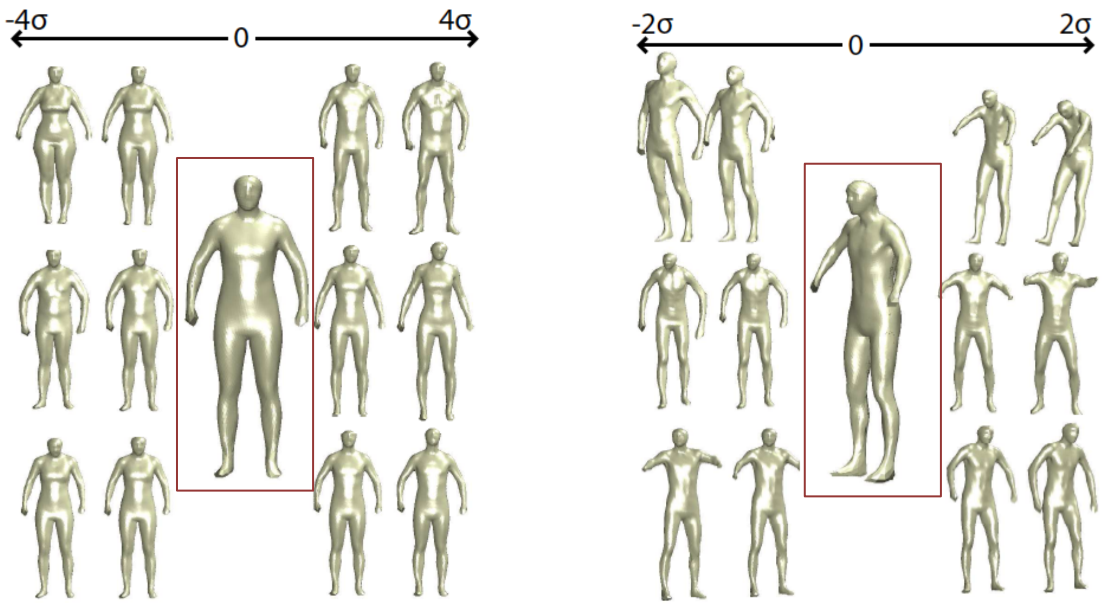
Archaeology



[Huang et al. 06]

Problem statement

- Given a population of 3D objects, we want to
  - Study and model the shape variability within the population (mean shape, modes of variation)



(a) Shape. (b) Pose.

Problem statement

- Given a population of 3D objects, we want to
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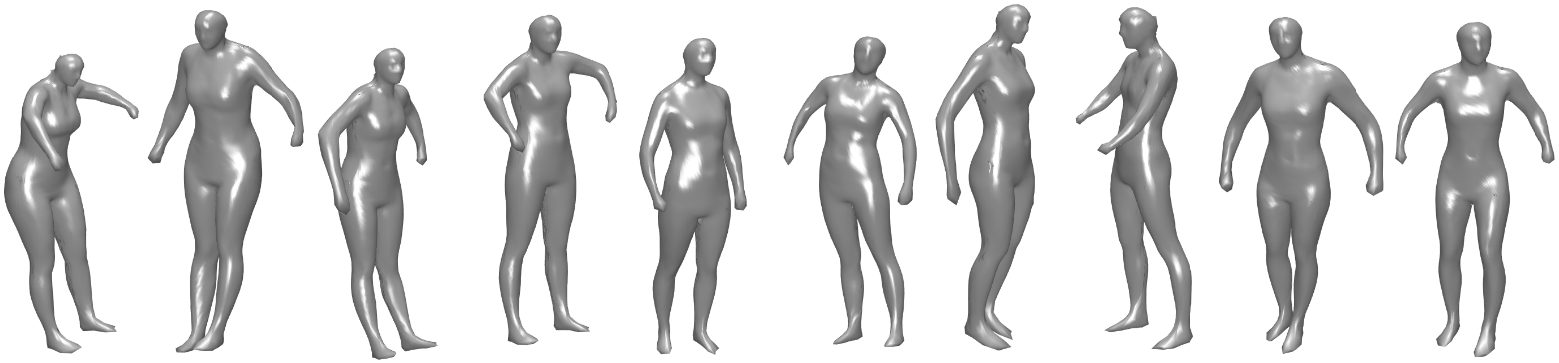
  - Build statistical models that describe the population

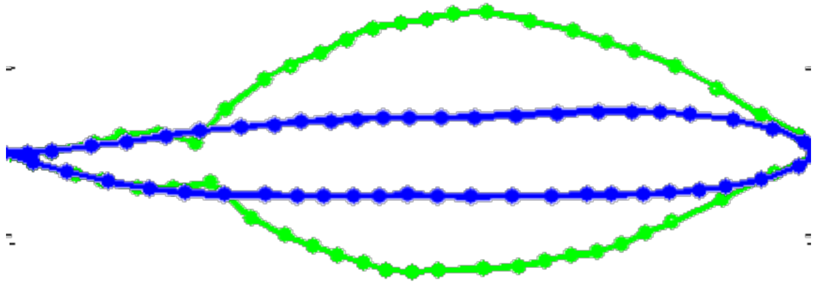


Automatically generated random human shapes in random poses

Problem statement

- Given a population of 3D objects, we want to
  - Study and model the shape variability within the population (mean shape, modes of variation)
  - Build statistical models that describe the population
  - Analyze & model deformations and growth patterns
    - How the 3D shape of the brain evolves with Alzheimer ?
    - Typical growth curve of a fœtus?
    - Differences in the growth of human body across countries
    - Correlation or causality relations (between the spatial distribution of Kebab shops with human body shape)

Let’s focus on the mean shape



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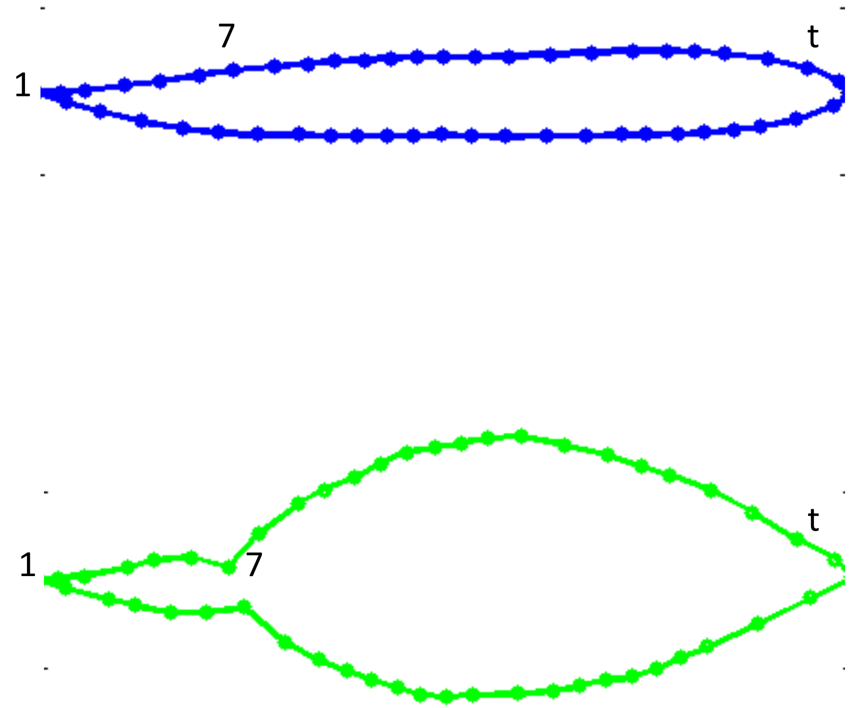
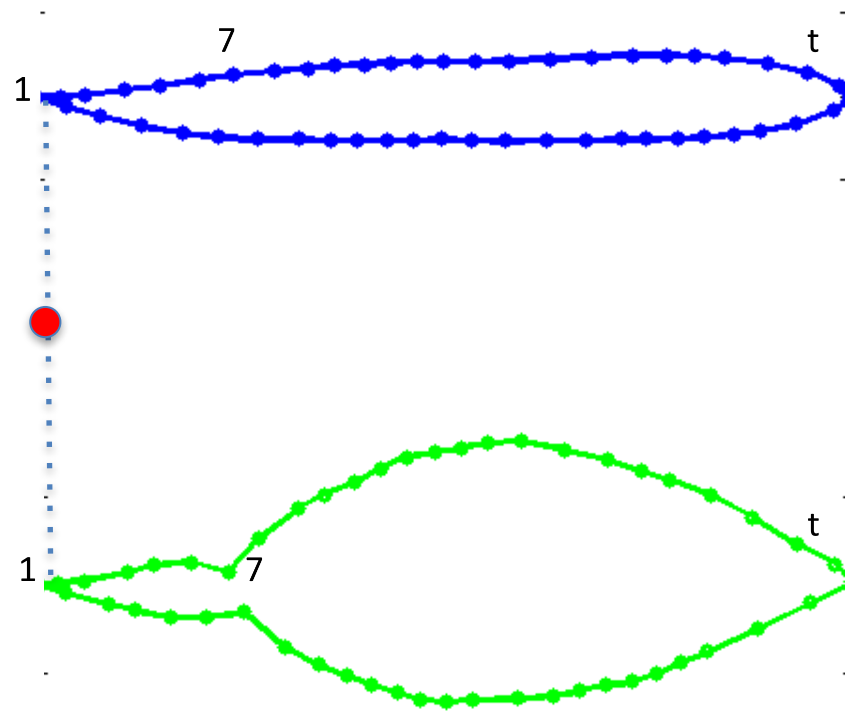


Let’s focus on the mean shape

- The registration problem



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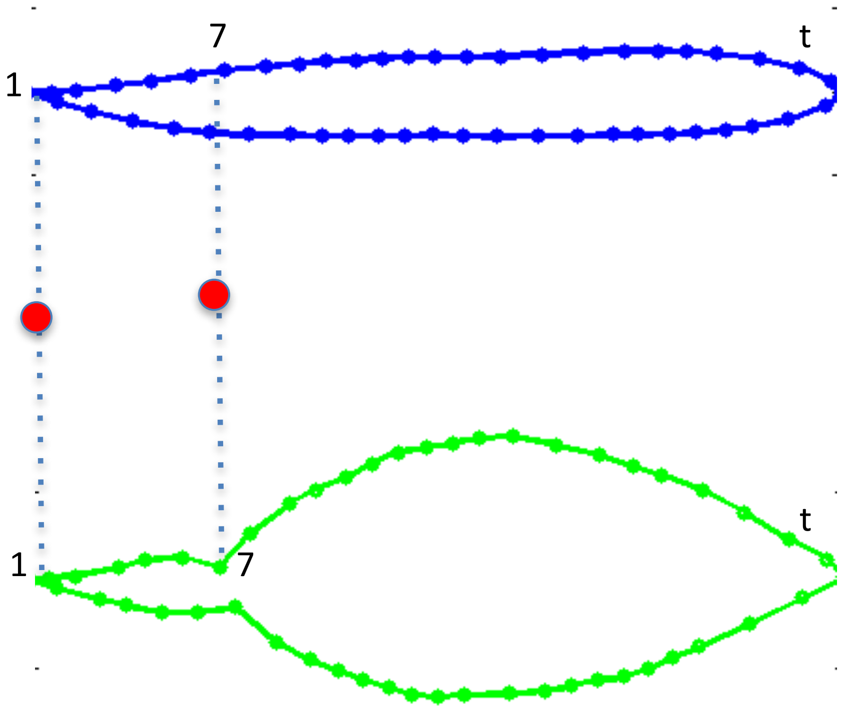

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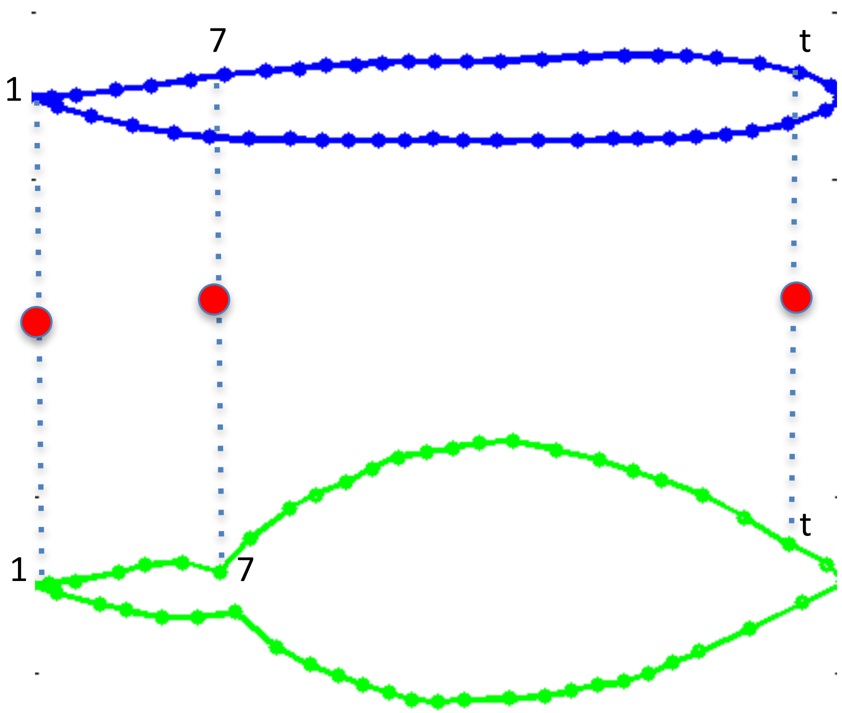

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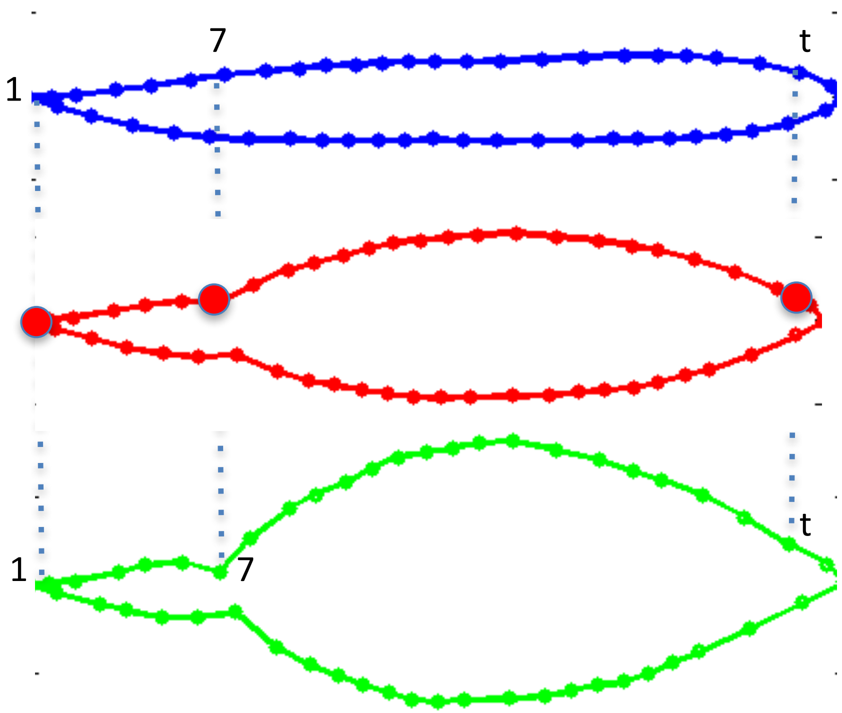
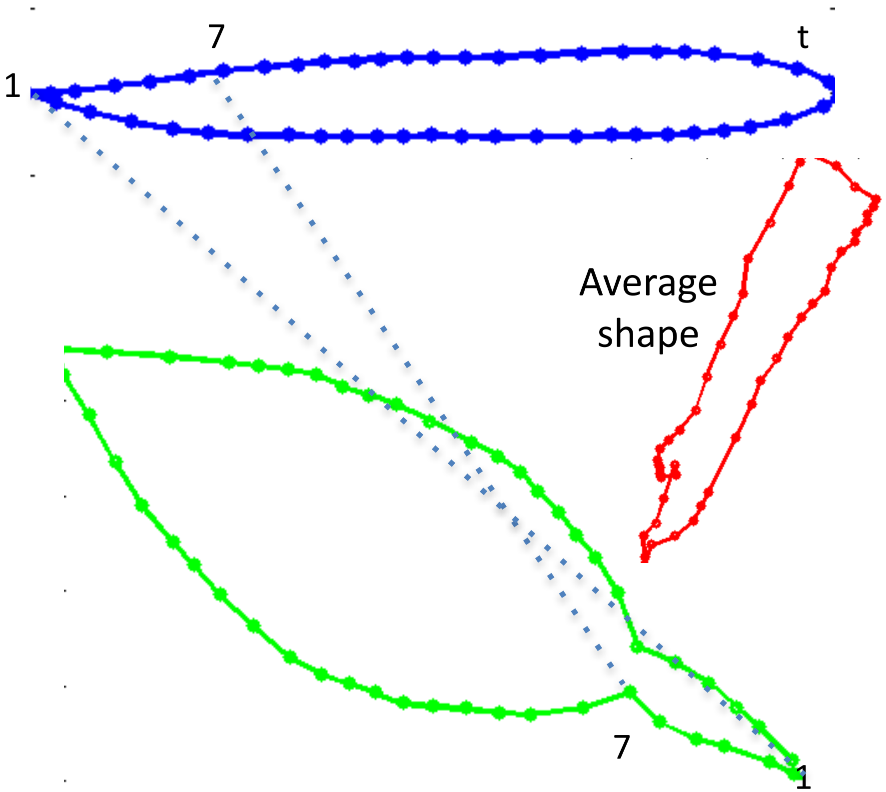


Things are not always so simple …

Wrong alignment
Correct correspondence



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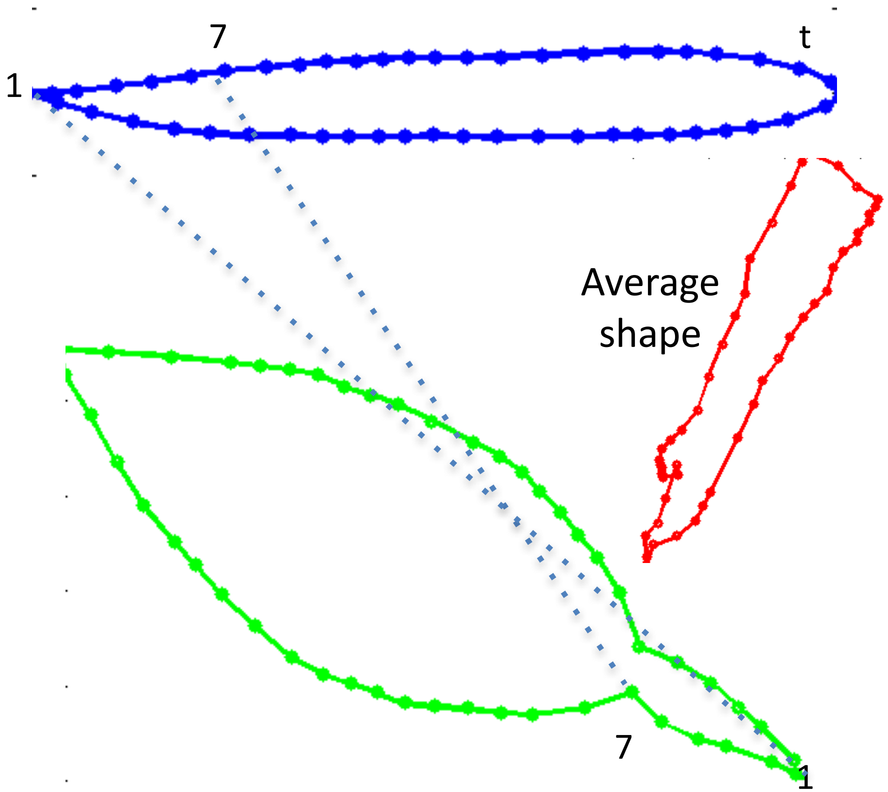
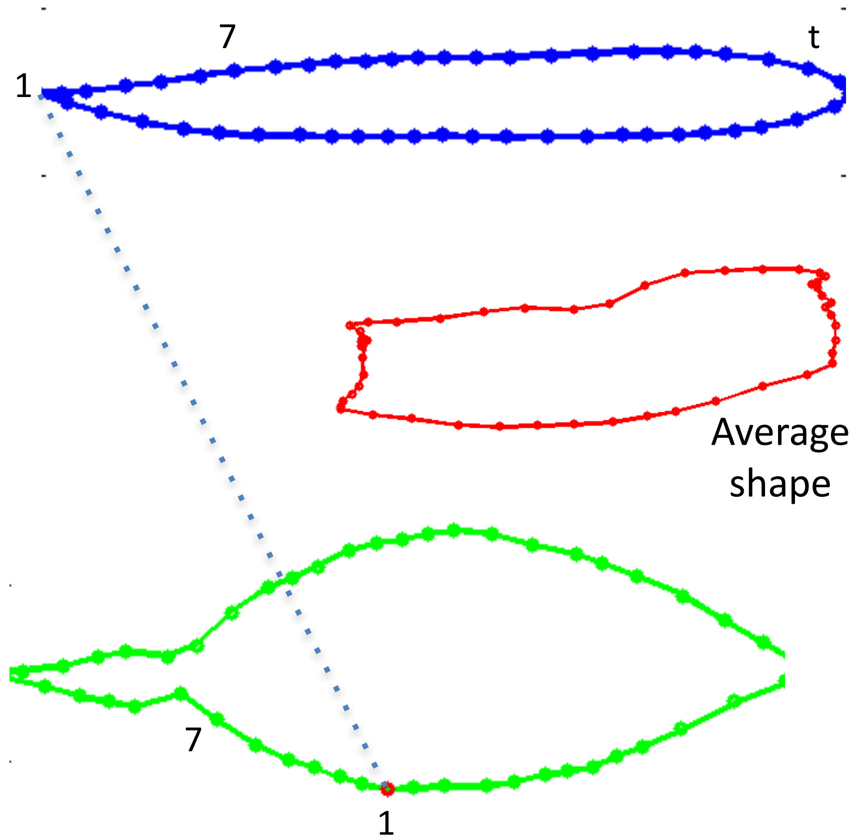
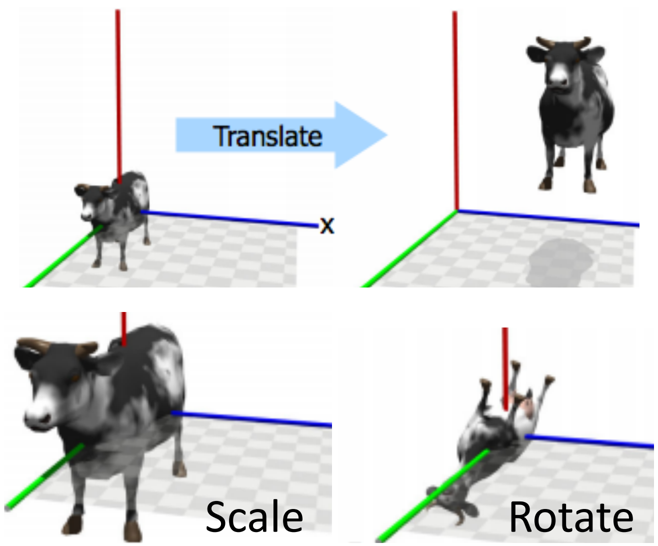


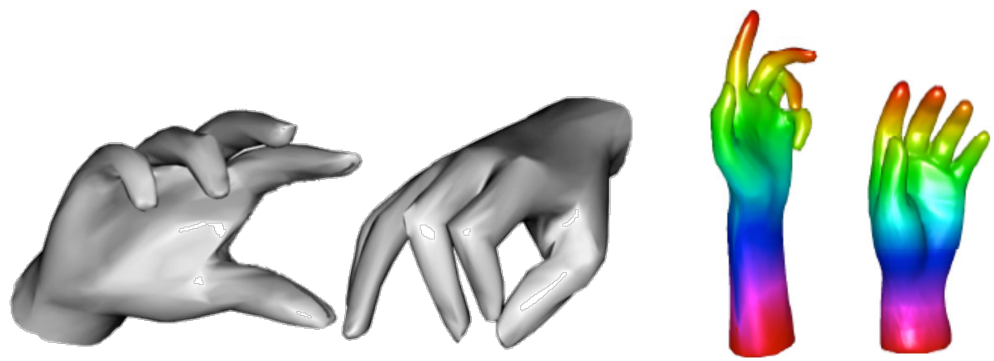
Correct alignment
Wrong correspondence



Let’s focus on the mean shape

- First problem - We need correct registration
  - Correct alignment and correct correspondences under rigid transformations & non-rigid deformations





Bending

Stretching

Rigid transformations
(do not change shape)

Non-rigid deformations
(do change the shape)

Let's focus on the mean shape

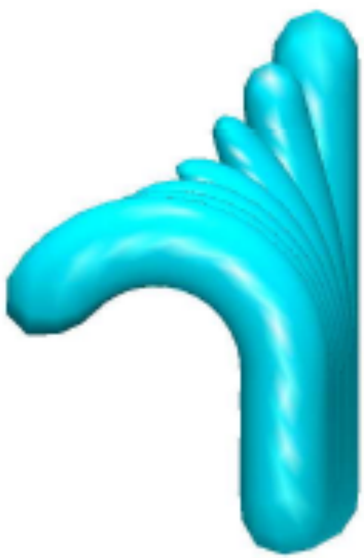
- The space of shapes is not Euclidean

(a) Linear path

$(1-t)f_1 + tf_2$

Let's focus on the mean shape

- The space of shapes is not Euclidean



(a) Linear path

$(1-t)f_1 + tf_2$

(b) Natural deformation

Let's focus on the mean shape

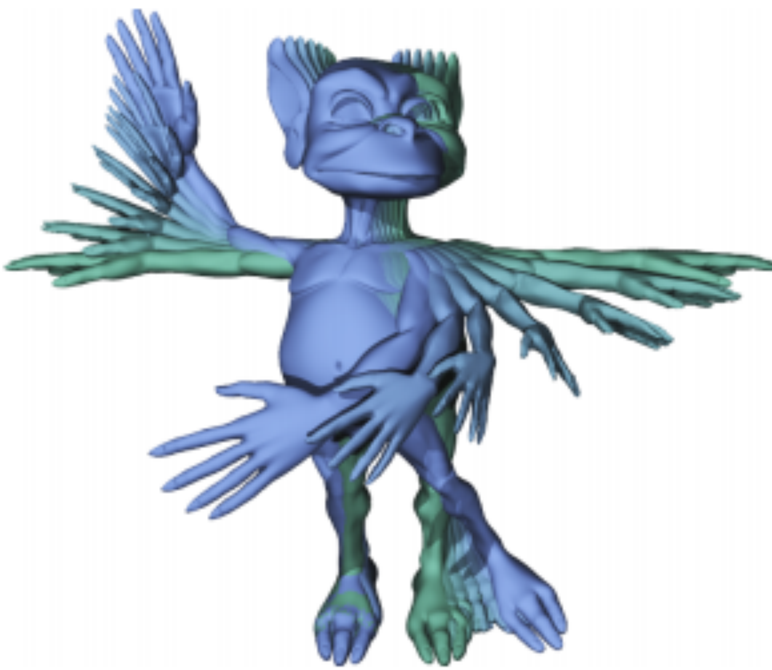
- The space of shapes is not Euclidean

(a) Linear path

$$(1-t)f_1 + tf_2$$

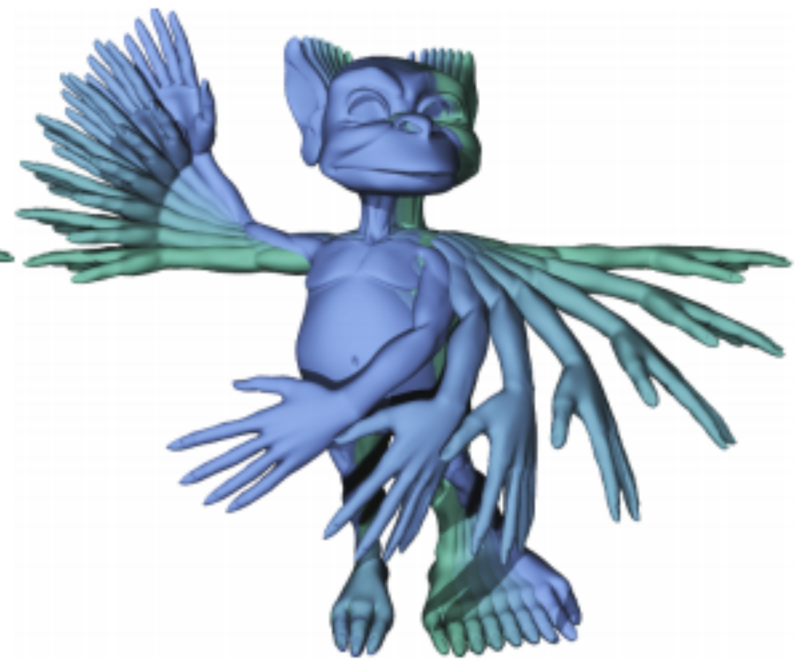
Let’s focus on the mean shape

- Second problem: we need an appropriate non-linear metric



(a) Linear path

$(1-t)f_1 + tf_2$



(b) Natural deformation

In this presentation

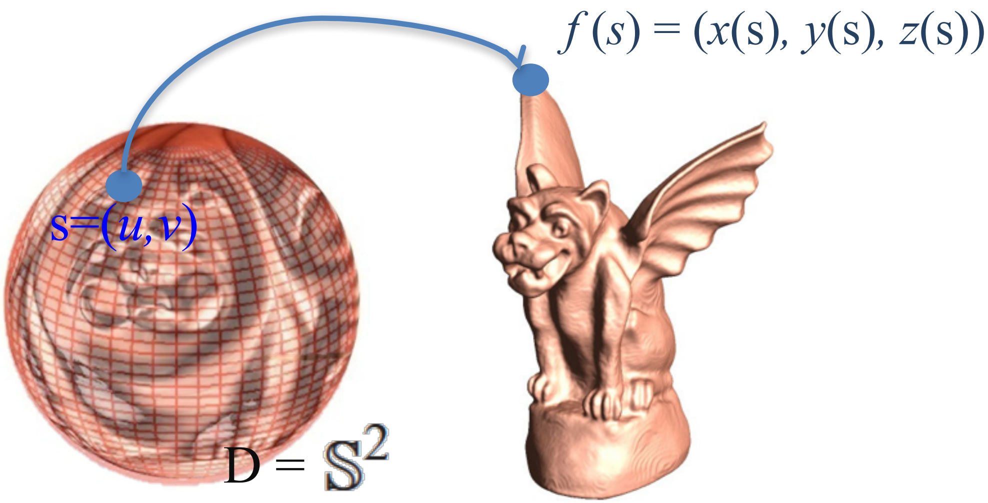
- Background and motivation
- Problem 1: Elastic registration
  - Surface representation
  - Re-parameterization and registration
- Problem 2: what is the right metric for comparing shapes
  - The elastic metric for 3D shape analysis
  - The Square Root Normal Field (SRNF) representation
- Problem 3: the SRNF inversion problem
- Applications

Representation of surfaces

- Parameterized surfaces

$$f : \mathbb{S}^2 \to \mathbb{R}^3$$

$\mathrm{s}=(u, v) \rightarrow f(s) = (x(\mathrm{s}), y(\mathrm{s}), z(\mathrm{s}))$



Spherical parameterization
of closed Genus-0 surfaces

Remove shape-preserving transformations

- Parameterized surfaces

$$f : \mathbb{S}^2 \to \mathbb{R}^3$$

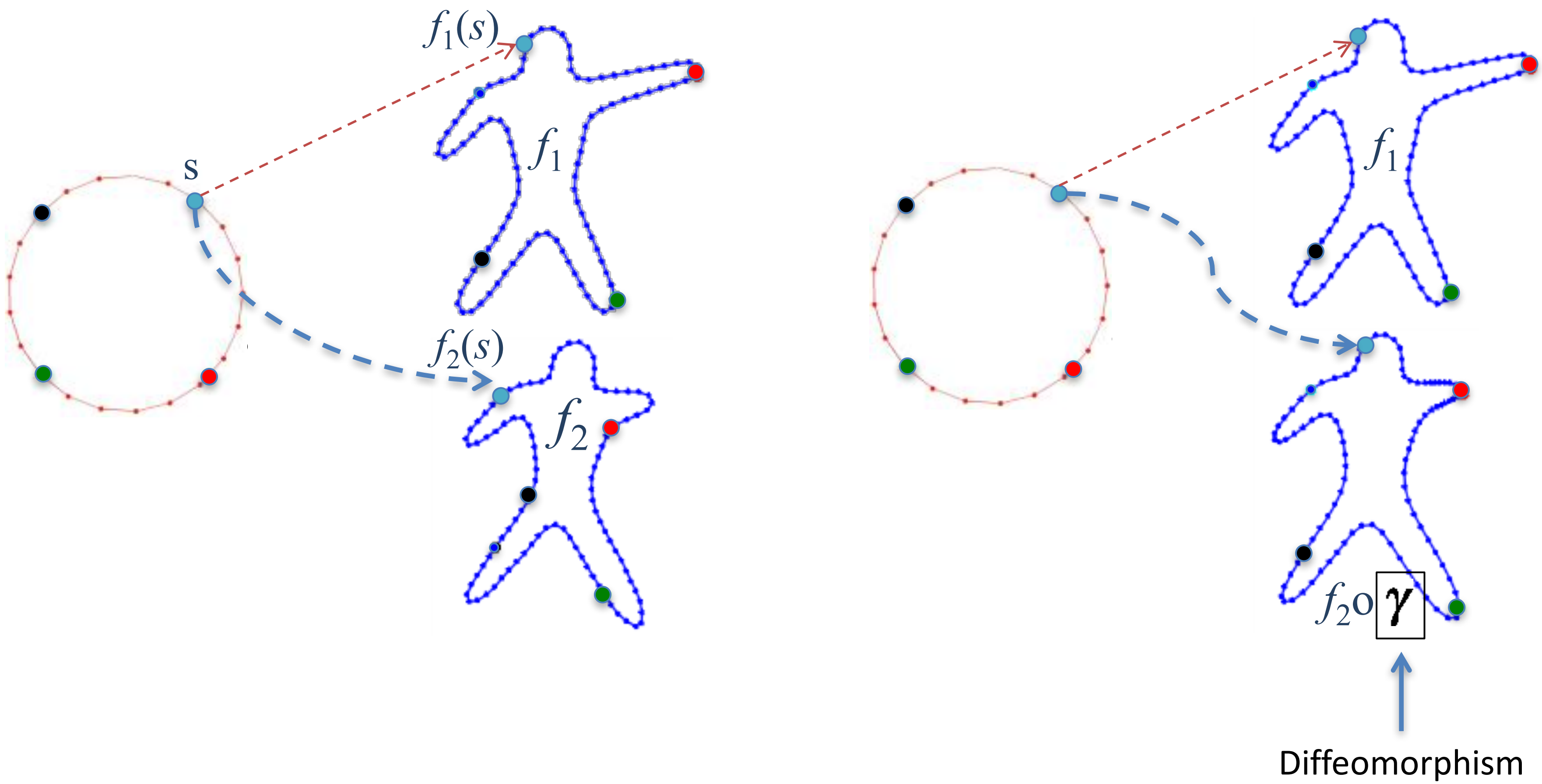
$$s=(u, v) \rightarrow f(s) = (x(s), y(s), z(s))$$

- Normalize all shapes for translation and scale
  - Translate all the shapes so that their centre of mass is at origin
  - Scale all the shapes to have unit surface area
- How about rotation and correspondences ?

Parameterization provides registration

Initial parameterization
(not optimal)

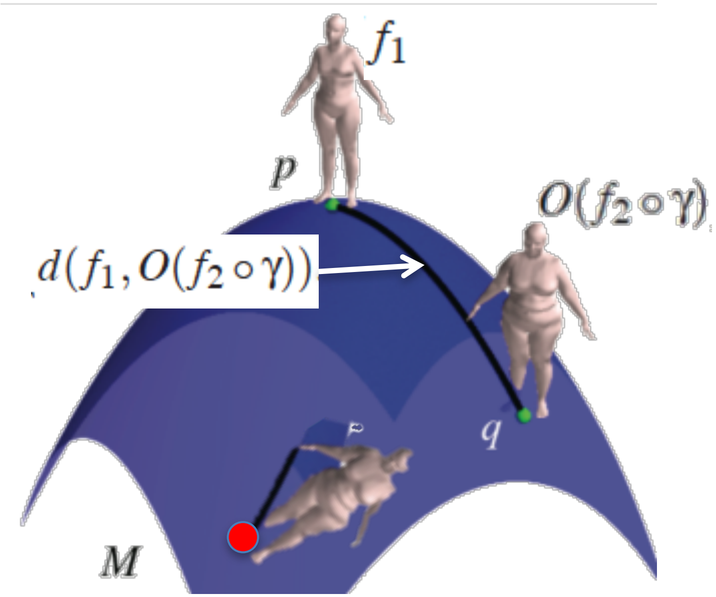
Re-parameterization of f_2
(optimal)



Parameterization provides registration

- Find the optimal rotation and diffeomorphism to apply to f2 so that it becomes as close as possible to f1

$$d_S([f_1],[f_2]) = \underbrace{\min_{O \in SO(3), \gamma \in \Gamma}}_{\text{Correspondence}} d(f_1, O(f_2 \circ \gamma)).$$



Let's focus on the mean shape

- First problem - We need correct registration

$$d_S([f_1],[f_2]) = \underbrace{\min_{O \in SO(3), \gamma \in \Gamma}}_{\text{Correspondence}} d(f_1, O(f_2 \circ \gamma)).$$

  - Assume correspondences are given
    - Find optimal rotation (SVD decomposition if d is Euclidean)
  - Assume optimal rotation is given
    - Solve for optimal corresondence, i.e. re-parameterization or diffeomorphism \gamma
    - Involves search over the space of all possible diffeomorphisms
  - Repeat many times the two steps above

Let's focus on the mean shape

- First problem - We need correct registration

$$d_S([f_1],[f_2]) = \underbrace{\min_{O \in SO(3), \gamma \in \Gamma}}_{\text{Correspondence}} d(f_1, O(f_2 \circ \gamma)).$$

- Second problem
  - What is *d* ? How do we measure distances between surfaces ?

In this presentation

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- Problem 1: Elastic registration
  – Surface representation
  – Re-parameterization and registration
- Problem 2: what is the right metric for comparing shapes
  – The elastic metric for 3D shape analysis
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The space of shapes is not Euclidean

(a) Linear path

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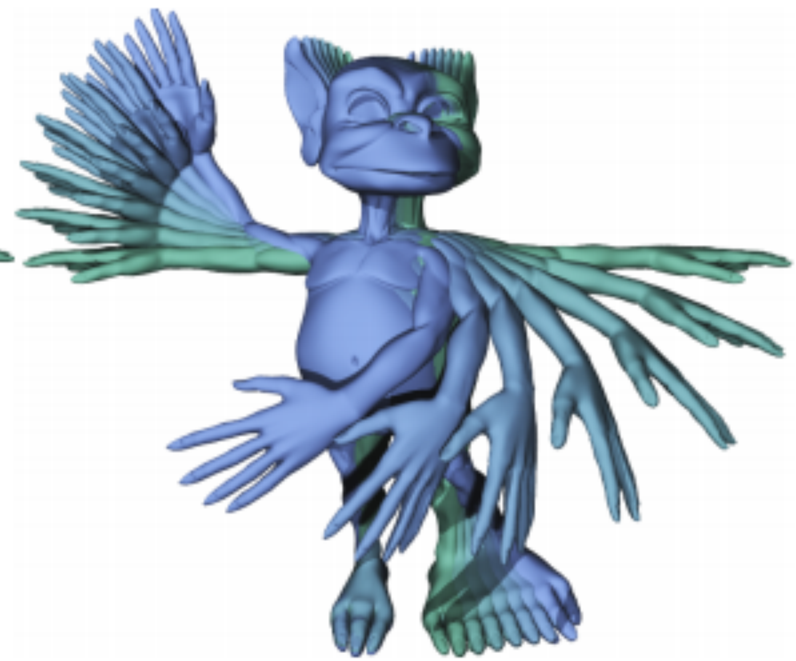


(b) Natural deformation

Elastic shape metric for comparing surfaces

The difference between two surfaces is the amount of bending and stretching needed to align one surface onto the other

Elastic shape metric for comparing surfaces

The difference between two surfaces is the amount of bending and stretching needed to align one surface onto the other

Three ways of quantifying bending

Differences in the orientation of normal vectors

Differences in the surface curvatures

Differences in the Second Fundamental Forms (II)

Elastic shape metric for comparing surfaces

The difference between two surfaces is the amount of bending and stretching needed to align one surface onto the other

Three ways of quantifying bending

Differences in the orientation of normal vectors

Differences in the surface curvatures

Differences in the Second Fundamental Forms (II)

Two ways of quantifying stretch (elasticity)

Differences in local surface area

Differences in the First Fundamental Form (the metric)

The general elastic metric

The difference between two surfaces is the amount of bending and stretching needed to align one surface onto the other

Three ways of quantifying bending

Differences in the orientation of normal vectors

Differences in the surface curvatures

Differences in the Second Fundamental Forms (II)

Two ways of quantifying strecth (elasticity)

Differences in local surface area

Differences in the First Fundamental Form (the metric)

Simplified elastic metric

The difference between two surfaces is the amount of bending and stretching needed to align one surface onto the other

Three ways of quantifying bending

Differences in the orientation of normal vectors

Differences in the surface curvatures

Differences in the Second Fundamental Forms (II)

Two ways of quantifying strecth (elasticity)

Differences in local surface area

Differences in the First Fundamental Form (the metric)

The partial elastic metric (Jermyn et al.)

- Surface bending
  - Change in the orientation of the normal vectors
  - Normal to a surface f at a point s

$$n(s) = \frac{\partial f}{\partial u}(s) \times \frac{\partial f}{\partial v}(s) \qquad \tilde{n}(s) = \frac{n(s)}{|n(s)|}$$

- Surface stretching
  - Change in local area of f at s: $r(s) = |n(s)|$

- A surface f can then be represented with $(r, \tilde{n})$

Partial elastic shape metric

The difference between f and g is the amount of bending and stretching needed to align one surface onto the other

$$d((r_1, \tilde{n}_1), (r_2, \tilde{n}_2)) = \alpha \int \frac{\partial r_1(s) \partial r_2(s)}{r(s)} ds + \beta \int \langle \partial \tilde{n}_1(s), \partial \tilde{n}_2(s) \rangle \, r(s) ds.$$

Elastic shape metric for comparing surfaces

The difference between f and g is the amount of bending and stretching needed to align one surface onto the other

Penalizes stretching

$$d((r_1, \tilde{n}_1), (r_2, \tilde{n}_2)) = \alpha \boxed{\int \frac{\partial r_1(s) \partial r_2(s)}{r(s)} ds} + \beta \int \langle \partial \tilde{n}_1(s), \partial \tilde{n}_2(s) \rangle \, r(s) ds.$$

Elastic shape metric for comparing surfaces

The difference between *f* and *g* is the amount of bending and stretching needed to align one surface onto the other

Penalizes stretching   Penalizes bending

$$d((r_1, \tilde{n}_1), (r_2, \tilde{n}_2)) = \alpha \int \frac{\partial r_1(s) \partial r_2(s)}{r(s)} ds + \beta \int \langle \partial \tilde{n}_1(s), \partial \tilde{n}_2(s) \rangle \, r(s) ds.$$

Elastic shape metric for comparing surfaces

The difference between f and g is the amount of bending and stretching needed to align one surface onto the other

- Not Euclidean
- Very complex to evaluate and use for shape statistics
- Computationally very expensive

Penalizes stretching — Penalizes bending

$$d((r_1, \tilde{n}_1), (r_2, \tilde{n}_2)) = \alpha \int \frac{\partial r_1(s) \partial r_2(s)}{r(s)} ds + \beta \int \langle \partial \tilde{n}_1(s), \partial \tilde{n}_2(s) \rangle \, r(s) ds.$$

Square-Root Normal Field (SRNF)

- SRNF representation of surfaces introduced by Jermyn et al. ECCV2012

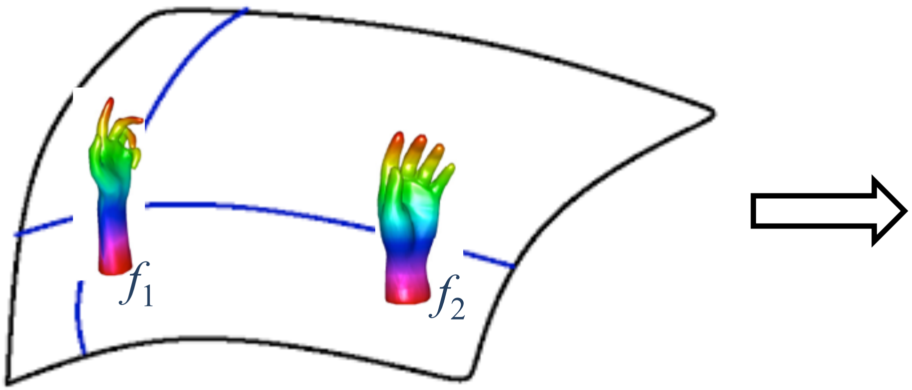
$$q(s) = \sqrt{r(s)}\tilde{n}(s) = \frac{n(s)}{\sqrt{r(s)}} = \frac{n(s)}{|n(s)|^{\frac{1}{2}}}.$$

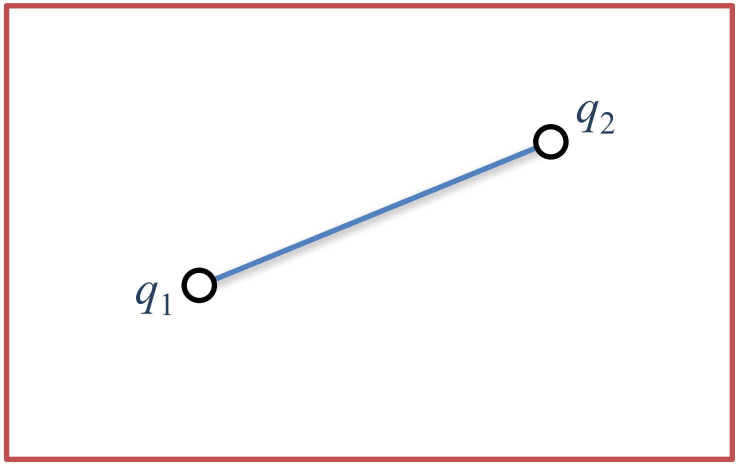
- For $\alpha = \frac{1}{4}$, and $\beta = 1$, the elastic metric reduces to the Euclidean distance between SRNF representations of surfaces

$$\begin{aligned} d(f_1, f_2) &= \frac{1}{4}\int \frac{\partial r_1(s)\partial r_2(s)}{r(s)} ds + \int \langle \partial\tilde{n}_1(s), \partial\tilde{n}_2(s)\rangle\, r(s) ds \\ &= \|q_1 - q_2\|^2 \\ &= \int \|q_1(s) - q_2(s)\|^2 ds. \end{aligned}$$

SRNF linearizes the manifold of shapes

Map all the shapes to the SRNF space





Space of parameterized surfaces (non-linear)

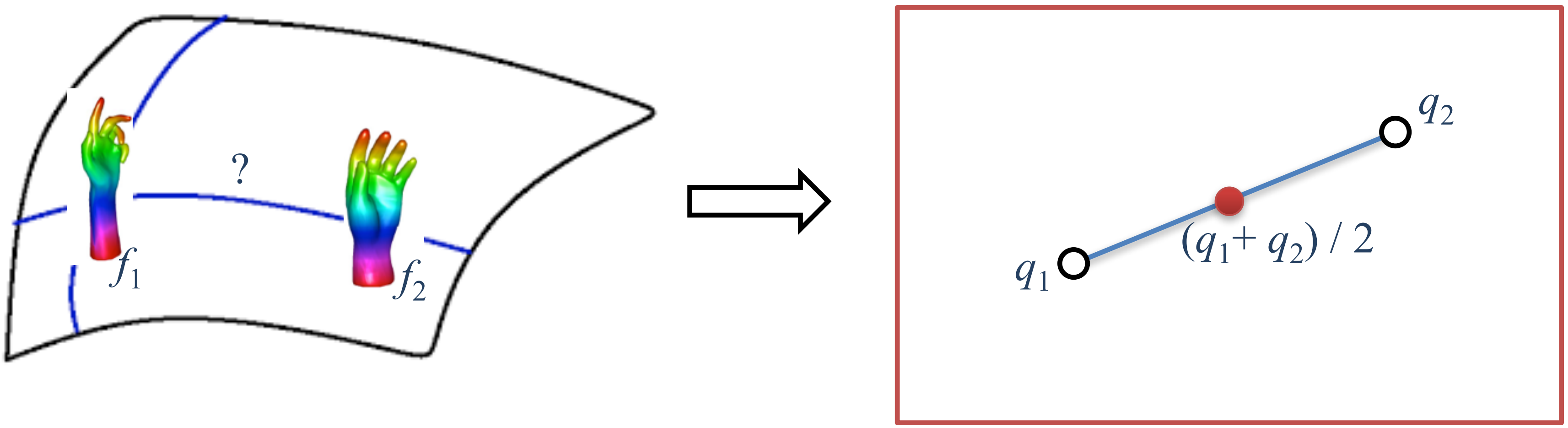
- Geodesic paths and distances are hard to compute
- Difficult to perform statistics

SRNF space is Euclidean

- Straight lines correspond to optimal deformations (geodesics)
- Standard linear statistics

SRNF linearizes the manifold of shapes

Perform all the analysis in the space of SRNFs



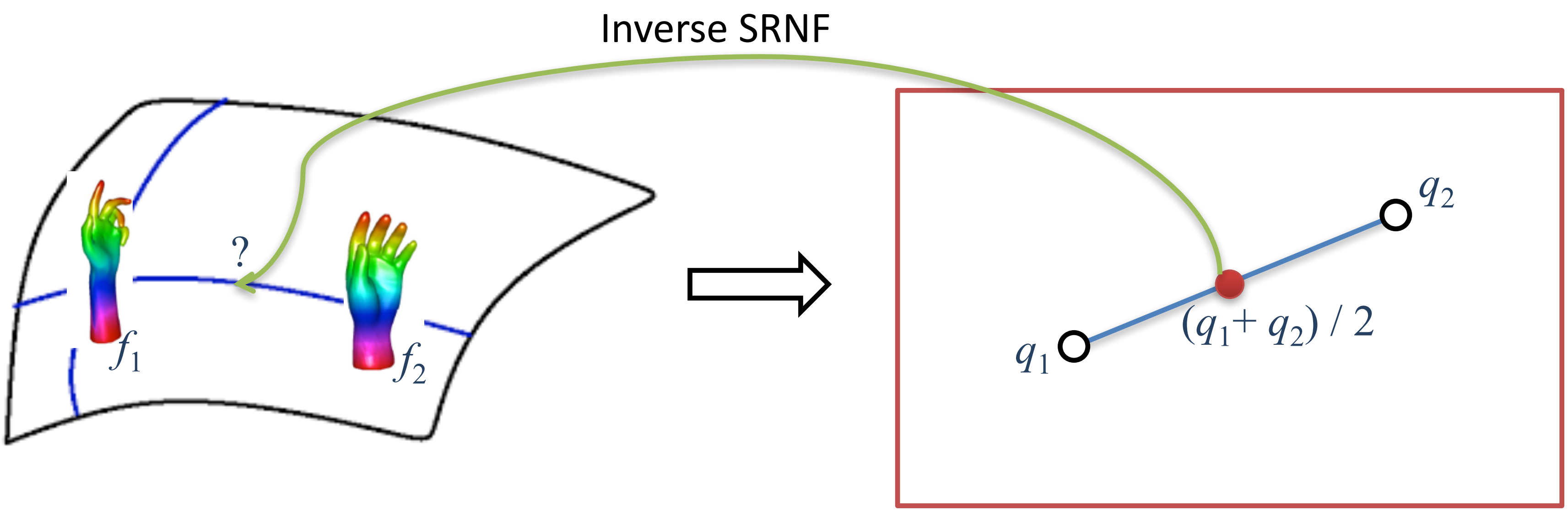
Space of parameterized surfaces (non-linear)

SRNF space is Euclidean

- Mean = $(q_1 + q_1) / 2$
- Linear interpolation: $tq_1 + (1-t)q_2$
- Statistics: standard PCA

SRNF linearizes the manifold of shapes

Map the results back to the space of surfaces



Space of parameterized surfaces (non-linear)

SRNF space is Euclidean

- Mean = $(q_1 + q_1) / 2$
- Linear interpolation: $tq_1 + (1-t)q_2$
- Statistics: standard PCA

Applications of SRNFs

- Comparison of shapes
- Elastic registration of shapes
- Computing geodesics (optimal deformation of one surface onto another)
- Transferring deformations
- Statistical shape analysis
  - Mean shape and modes of variations
  - Characterizing populations with probability distributions
  - Generating arbitrary 3D shapes

Using the SRNF

- Comparing 3D shapes

$$
\begin{aligned}
d(f_1, f_2) &= \min_{O \in SO(3), \gamma \in \Gamma} d(f_1 - O(f_2 \circ \gamma)) \\
&= \min_{O \in SO(3), \gamma \in \Gamma} \|q_1 - O(q_2, \gamma)\|
\end{aligned}
$$

$$
O(q_2, \gamma) = (q_2 \circ \gamma)\sqrt{J_\gamma}
$$

where J_γ is the determinant of the Jacobian of γ.

Using the SRNF

- Comparing 3D shapes

$$
\begin{aligned}
d(f_1, f_2) &= \min_{O \in SO(3), \gamma \in \Gamma} d(f_1 - O(f_2 \circ \gamma)) \\
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\end{aligned}
$$

$$O(q_2, \gamma) = (q_2 \circ \gamma)\sqrt{J_\gamma}$$

where J_γ is the determinant of the Jacobian of γ.

- Elastic registration of 3D shapes

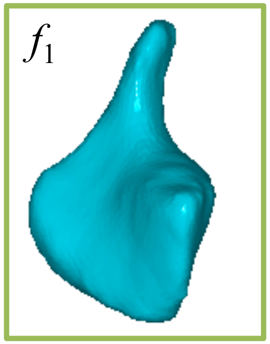
$$
\begin{aligned}
(O^*, \gamma^*) &= \operatorname*{arg\,min}_{O \in SO(3), \gamma \in \Gamma} d(f_1 - O(f_2 \circ \gamma)) \\
&= \operatorname*{arg\,min}_{O \in SO(3), \gamma \in \Gamma} \|q_1 - O(q_2, \gamma)\|
\end{aligned}
$$

Registration and classification results

- Registration results
  - Anatomical surfaces
  - Complex shapes
- Classification results
  - Generic 3D shapes (SHREC07 dataset)
  - Medical imaging
    - diagnosis of attention deficit hyperactivity disorder (ADHD)

Registration results – anatomical 3D shapes

Elastic registration of carpal bones



Registration results – anatomical 3D shapes

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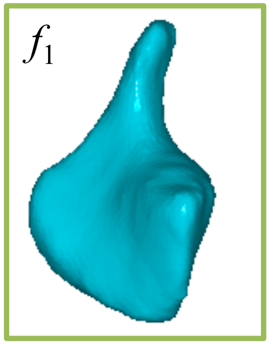

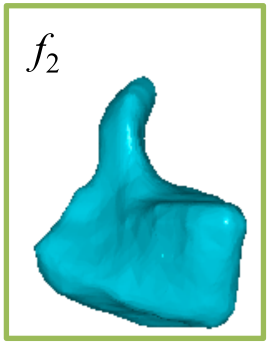

Registration results – anatomical 3D shapes

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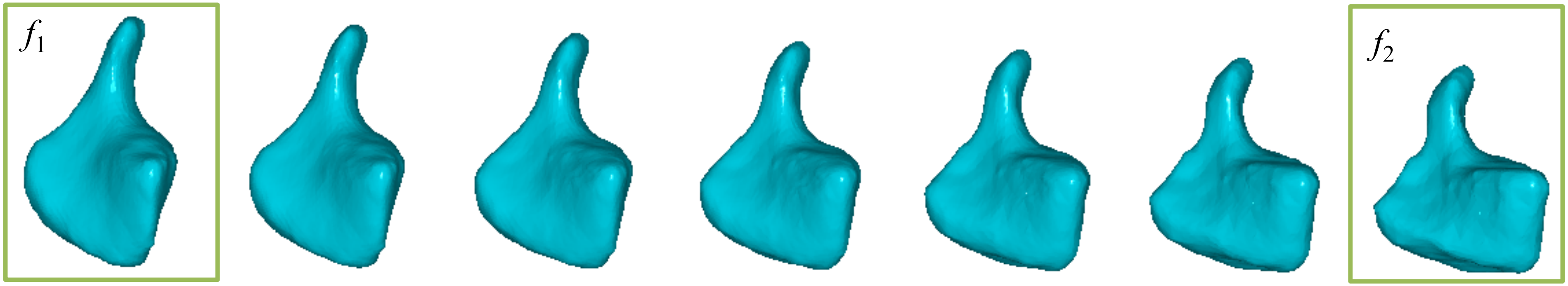

Registration results – anatomical 3D shapes

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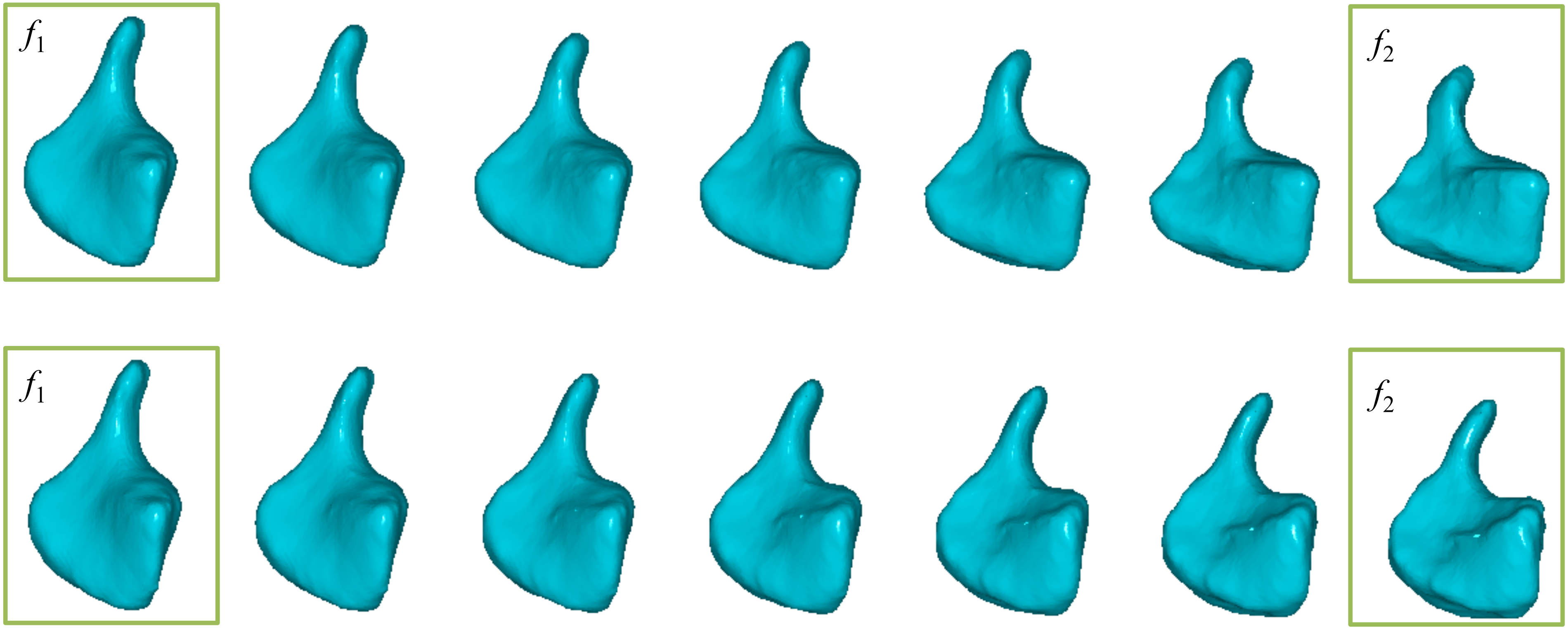
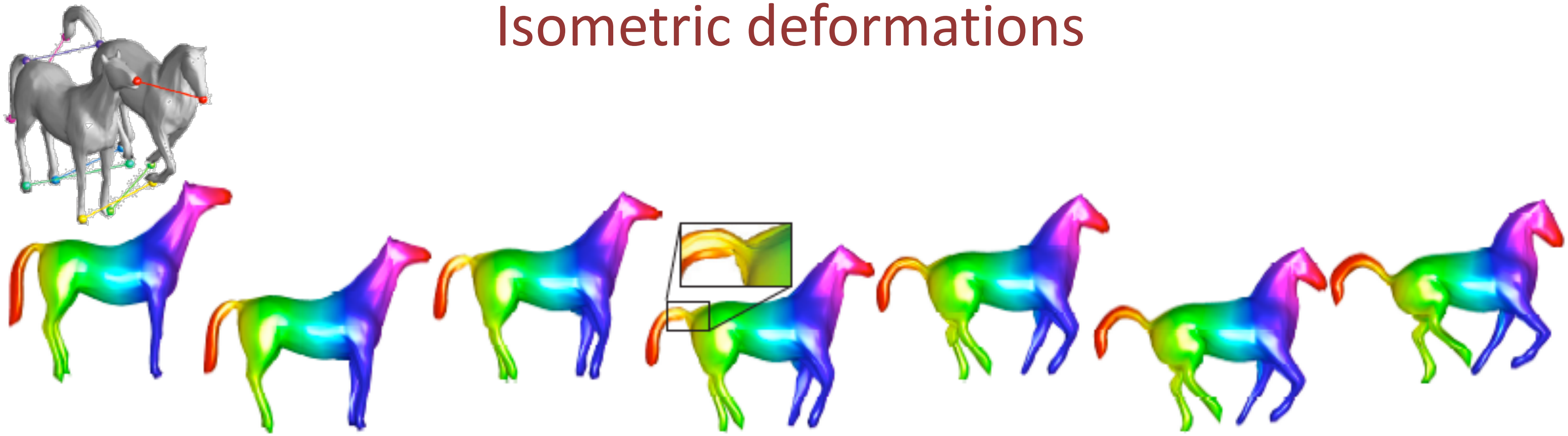
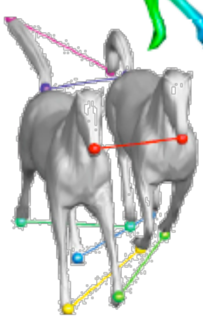


Correspondence results – complex shapes

Isometric deformations



$$L[F^*] = 0.1609$$

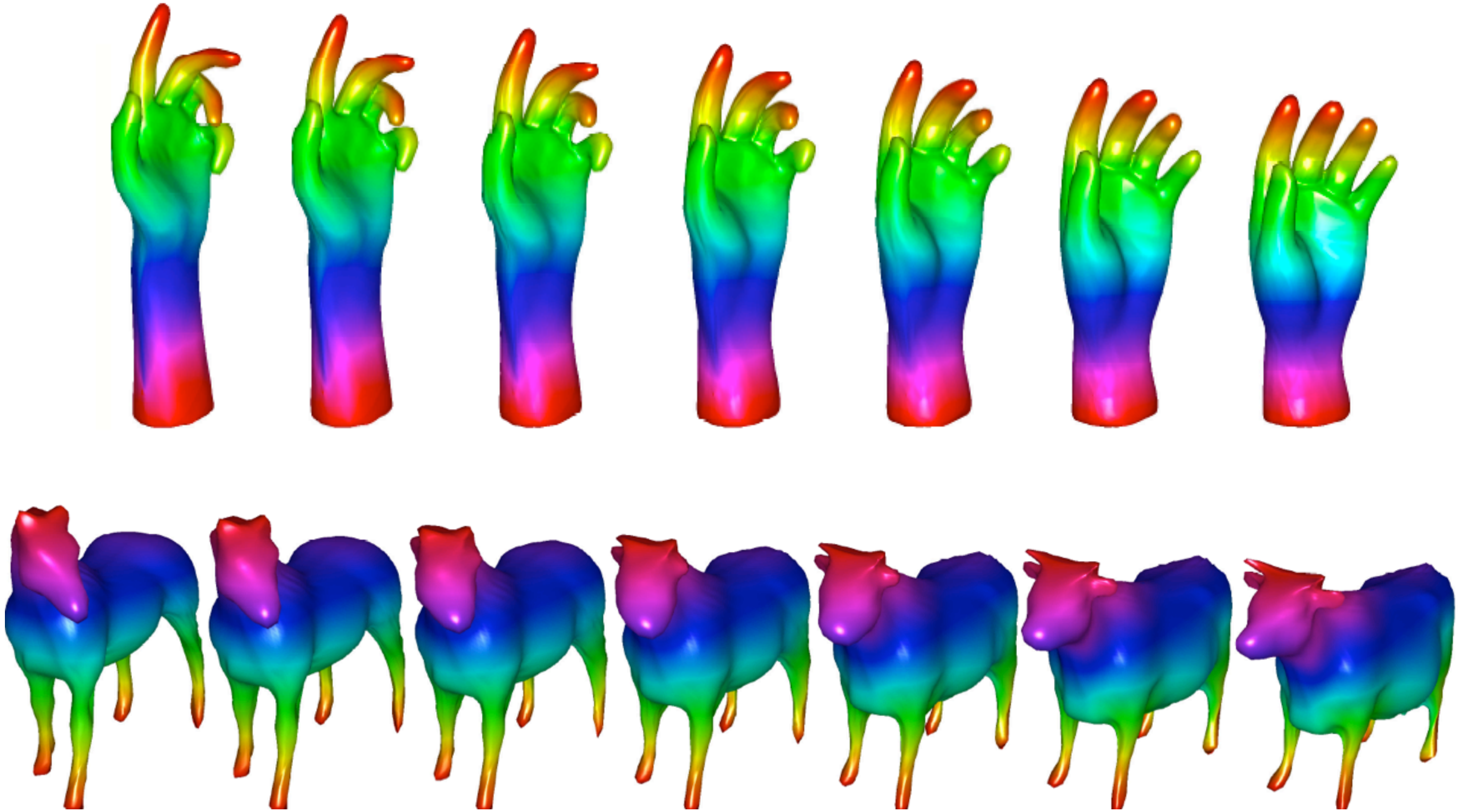


$$L[F^*] = 0.1369$$

Correspondences are color-coded

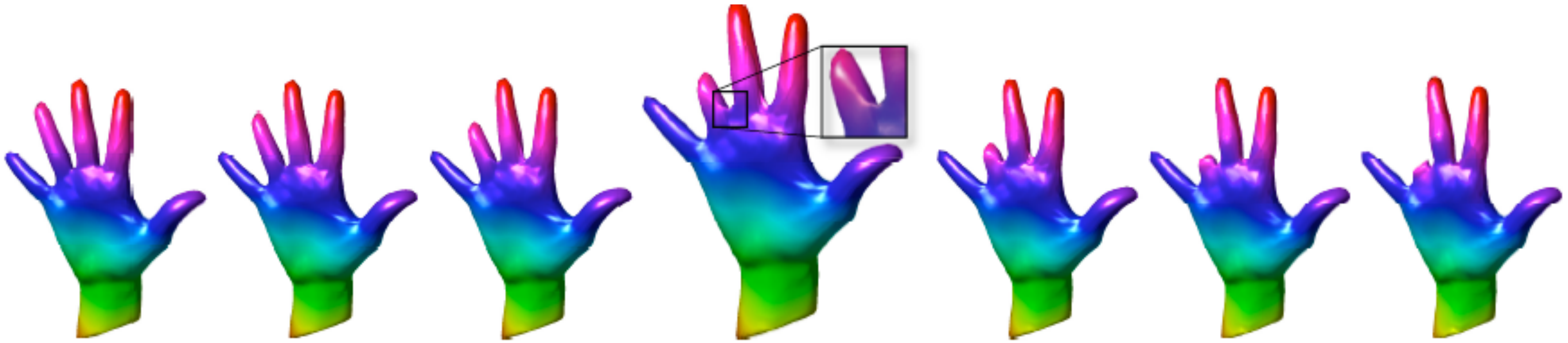
Correspondence results – complex shapes

Elastic deformations



Correspondence results – complex shapes

Correspondence in the presence of missing parts



$L[F^*] = 0.0997$

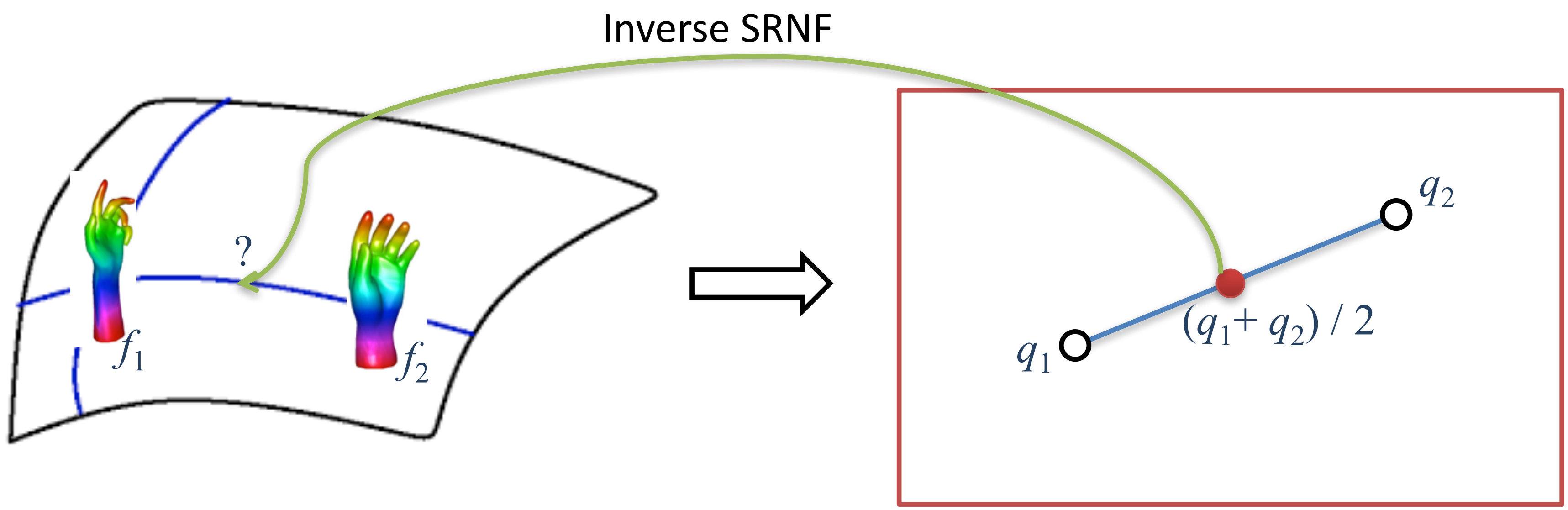
$(L[F^*] = 0.1977)$

In this presentation

- Background and motivation
- Problem 1: Elastic registration
  - Surface representation
  - Re-parameterization and registration
- Problem 2: what is the right metric for comparing shapes
  - The elastic metric for 3D shape analysis
  - The Square Root Normal Field (SRNF) representation
- Problem 3: the SRNF inversion problem
- Applications

Shape statistics using SRNFs

- The SRNF map should be invertible



Space of parameterized surfaces (non-linear)

SRNF space (linear)

- Mean = $(q_1 + q_1) / 2$
- Linear interpolation: $tq_1 + (1-t)q_2$
- Statistic: standard PCA

SRNF maps inversion

- The SRNF map should be invertible
  - We know how to compute SRNFs

$$q(s) = \sqrt{r(s)}\tilde{n}(s) = \frac{n(s)}{\sqrt{r(s)}} = \frac{n(s)}{|n(s)|^{\frac{1}{2}}}.$$

  - Their inverse is not unique and it does not have a closed analytical form (at least we don't know it)
- Good news
  - We can invert it numerically

SRNF inversion

- Formulation
  - Given q, we want to find *f* such that SRNF(*f*) = Q(*f*) is as close as possible to q

$$E_0(f;q) = \min_{O,\gamma} \|Q(f) - O(q,\gamma)\|_2^2$$

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  - Define the surface *f* as the deformation of a reference surfsce f_0 (e.g. a sphere)

$$f = f_0 + w,$$

  - Parameterize the space of deformations with some orthonormal basis

$$w = \sum_{b \in \mathcal{B}} \alpha_b b,$$

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$$f = f_0 + w,$$

  - Parameterize the space of deformations with some orthonormal basis

$$w = \sum_{b \in \mathcal{B}} \alpha_b b,$$

- General surfaces: spherical harmonic basis
- Domain-specific data: use PCA basis

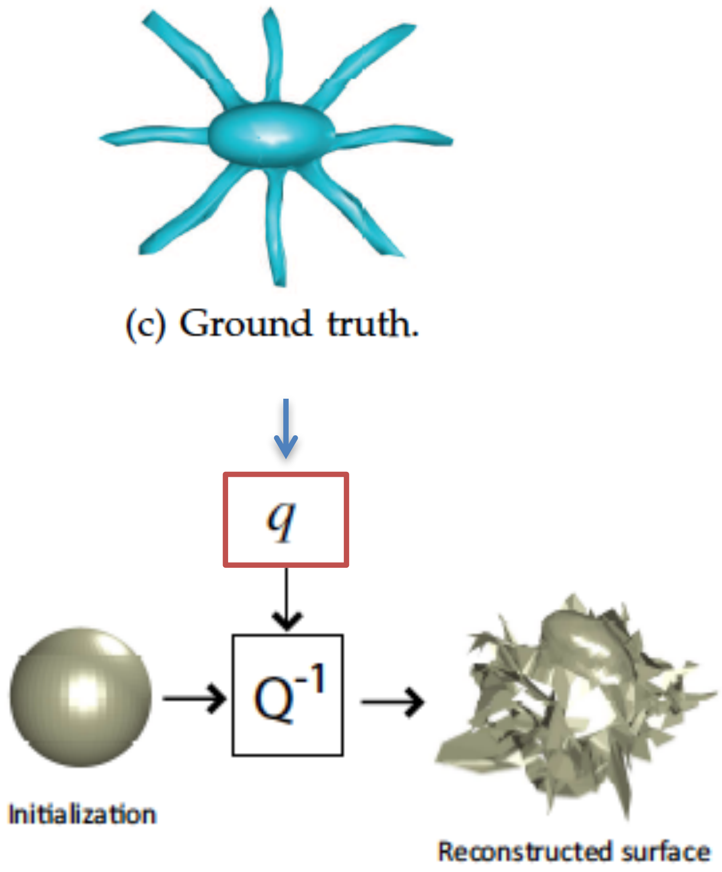
SRNF inversion

- Formulation
  - Given q, we want to find f such that SRNF(f) = Q(f) is as close as possible to q

$$E_0(f; q) = \min_{O,\gamma} \|Q(f) - O(q,\gamma)\|_2^2$$

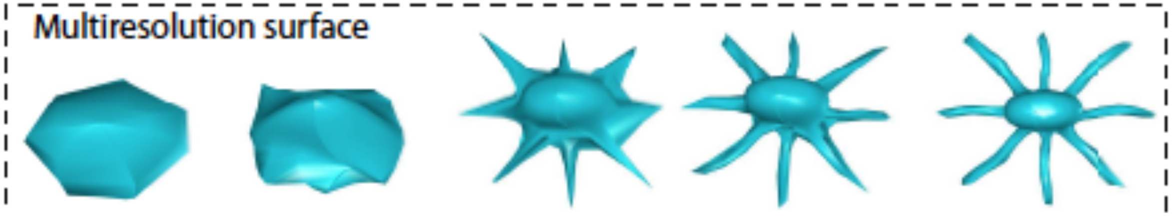
$$E(w; q) = \min_{O,\gamma} \|Q(f_0 + w) - O(q,\gamma)\|_2^2 \ ,$$

SRNF inversion by gradient descent



Multi-resolution SRNF representation

- Use spherical wavelet decomposition



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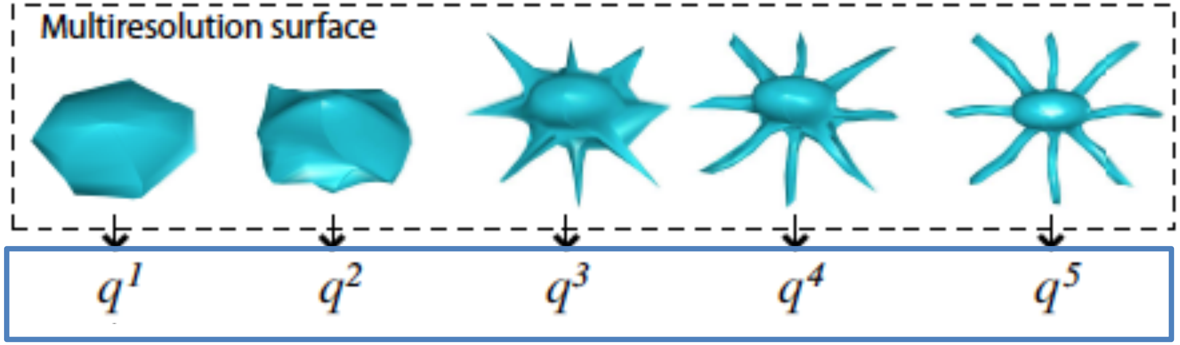


Multiresolution SRNF

Multi-resolution SRNF representation

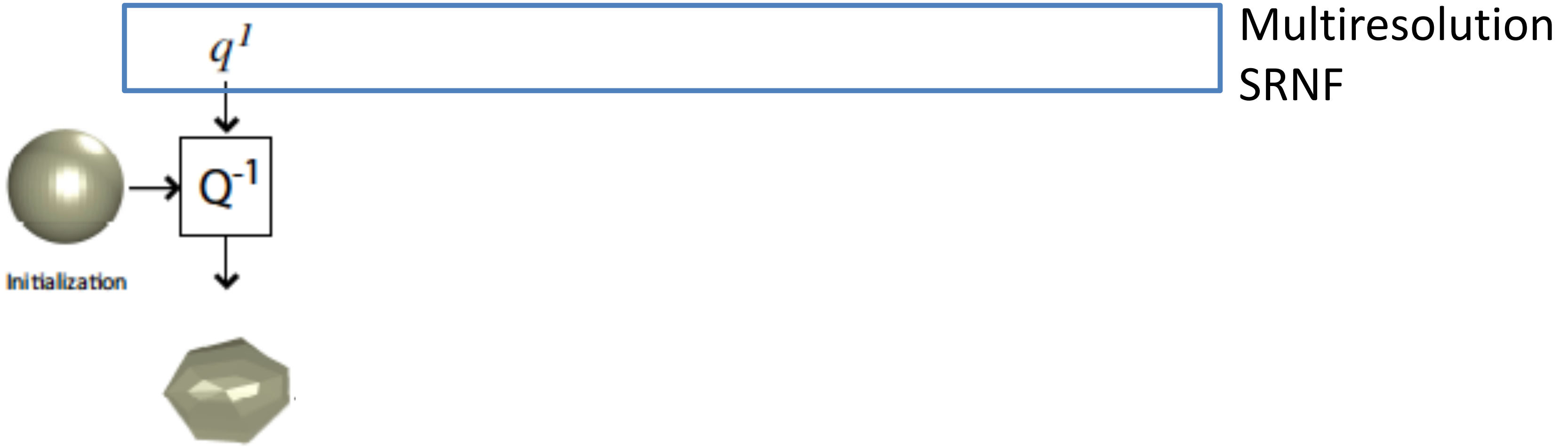
- Use spherical wavelet decomposition

q^1	q^2	q^3	q^4	q^5

Multiresolution SRNF

Multi-resolution SRNF representation

- Use spherical wavelet decomposition



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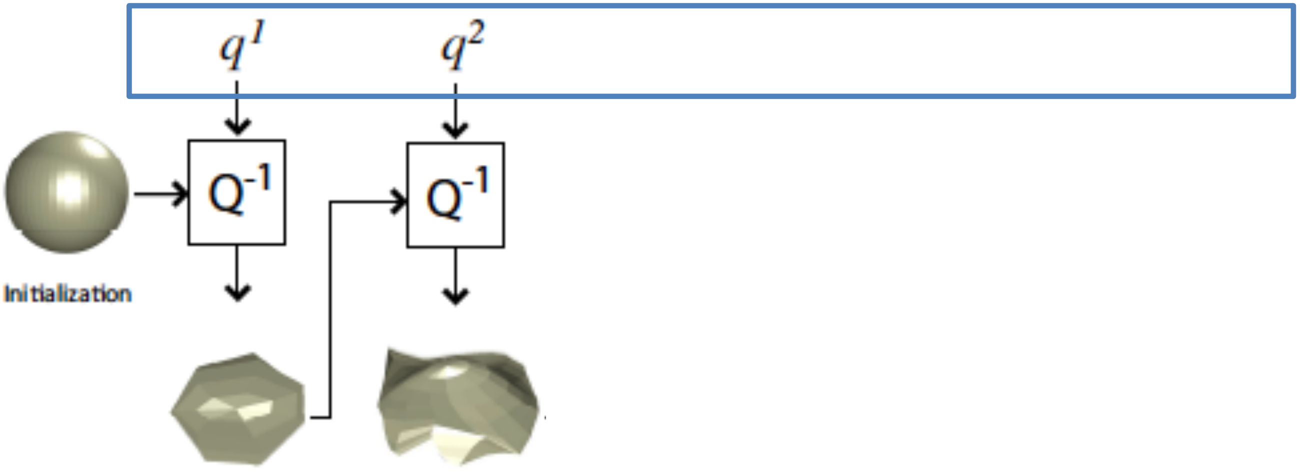
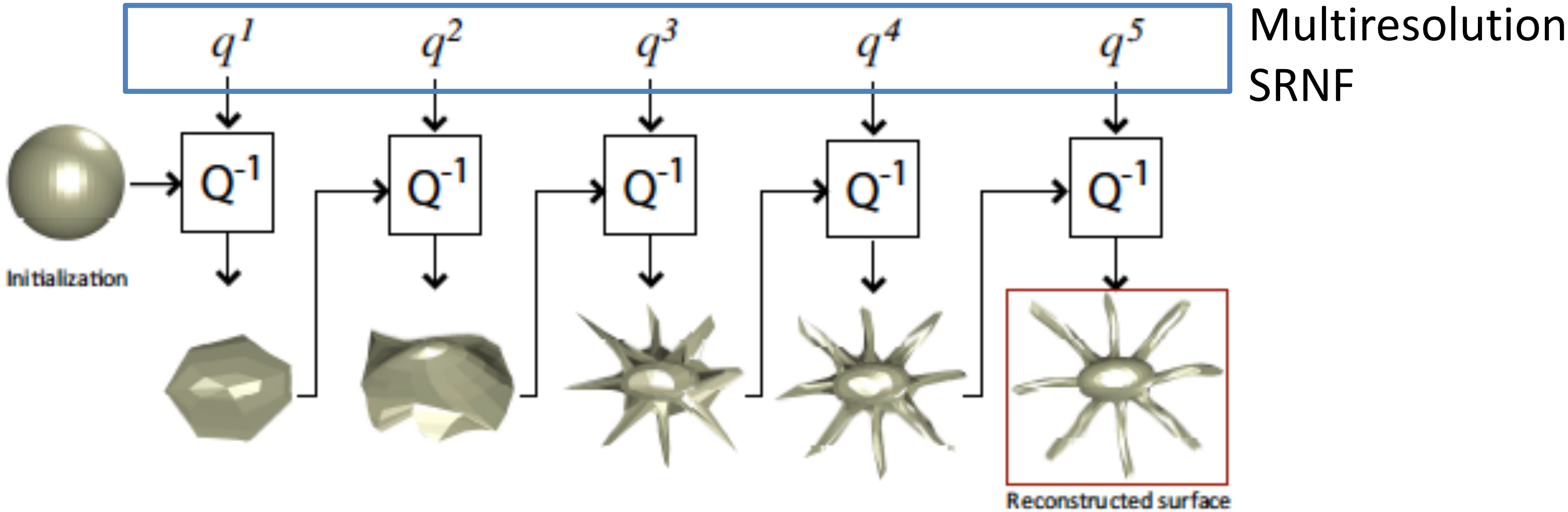


Multiresolution
SRNF

Multi-resolution SRNF representation

- Use spherical wavelet decomposition



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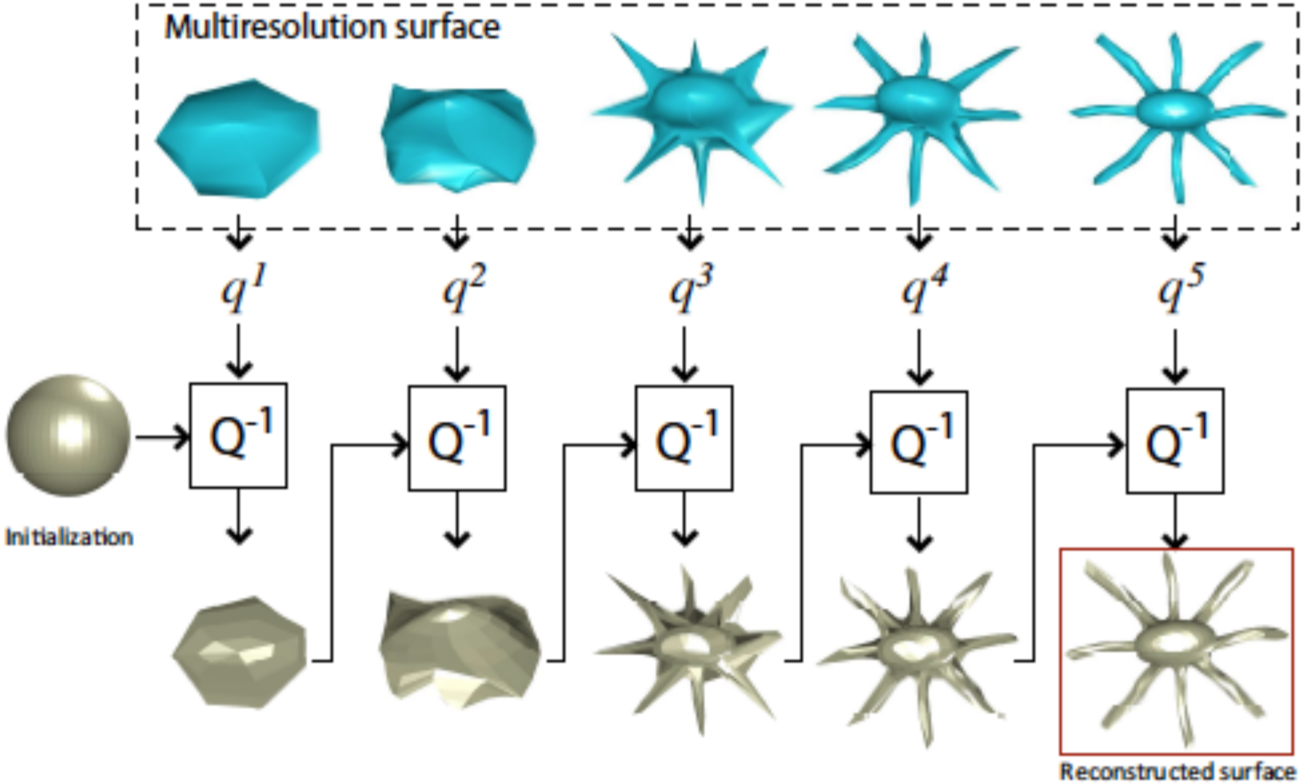
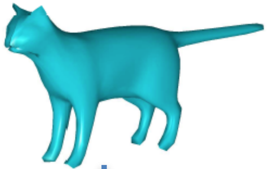
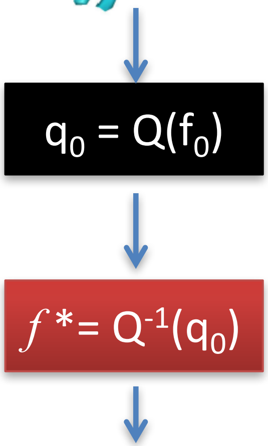
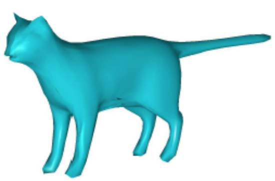


Some inversion results

(a) The target surfaces f_o.





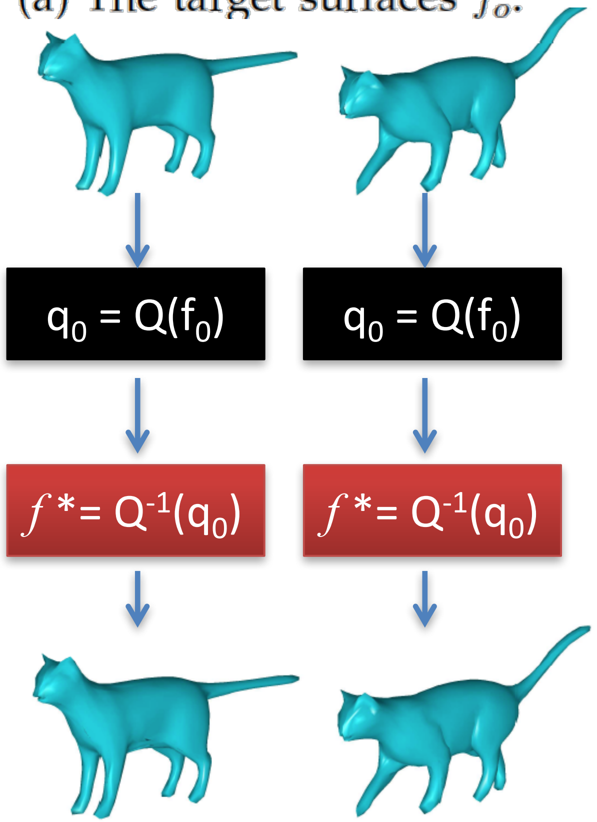


(b) The reconstructed surfaces f^*.

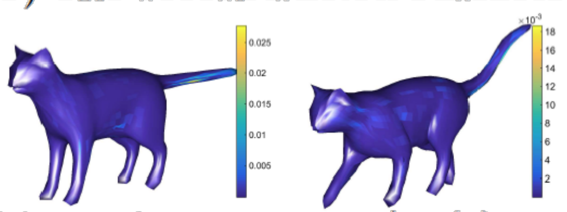
(c) Pixel-wise errors $|f^*(s) - f_o(s)|$.

Some inversion results

(a) The target surfaces f_o.



(b) The reconstructed surfaces f^*.



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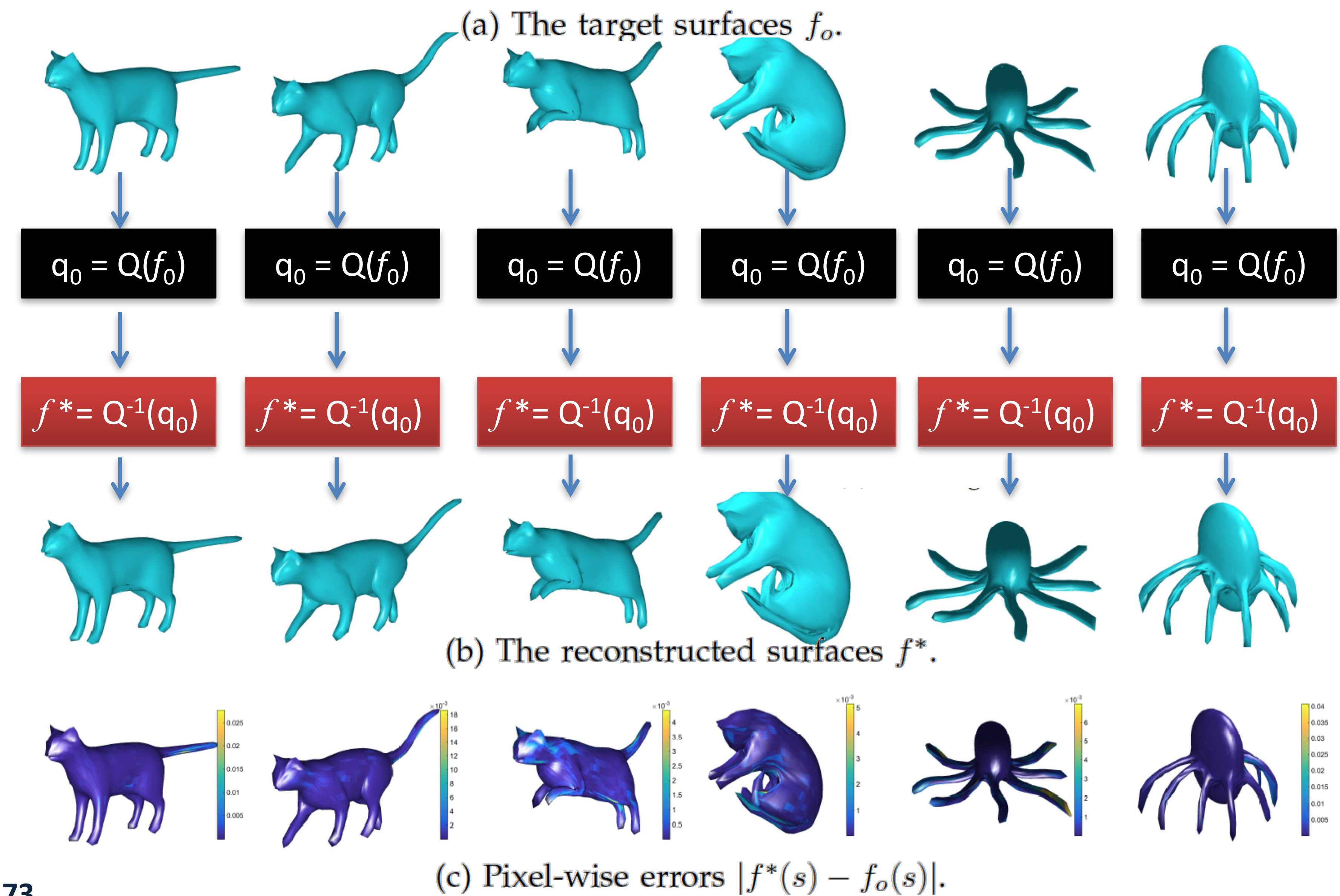
Some inversion results



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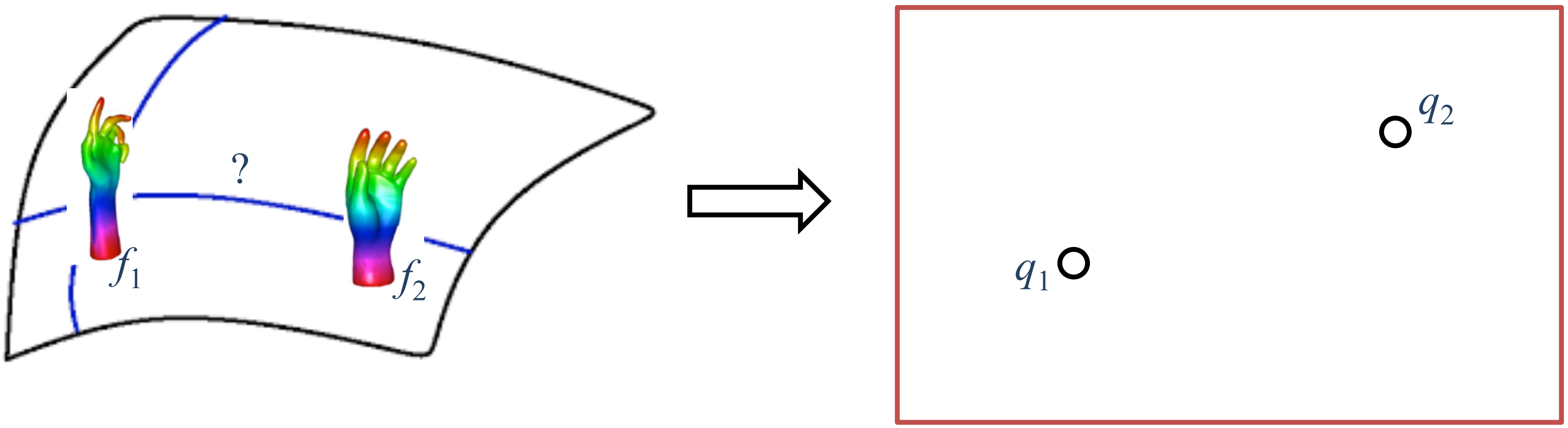
Geodesic paths

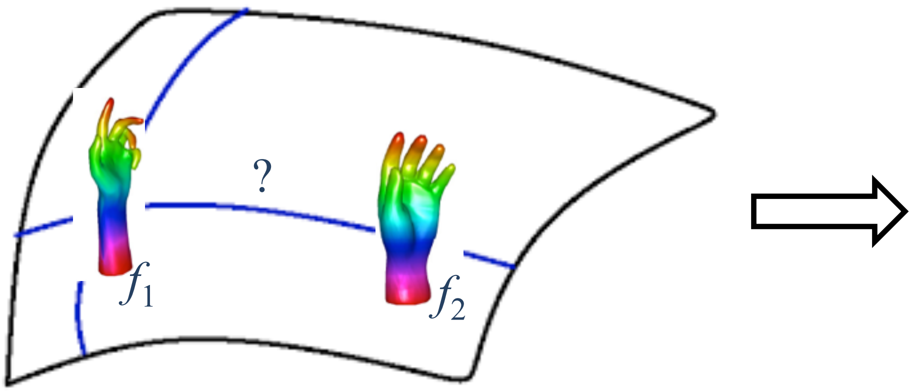
- Map shapes to SRNF space

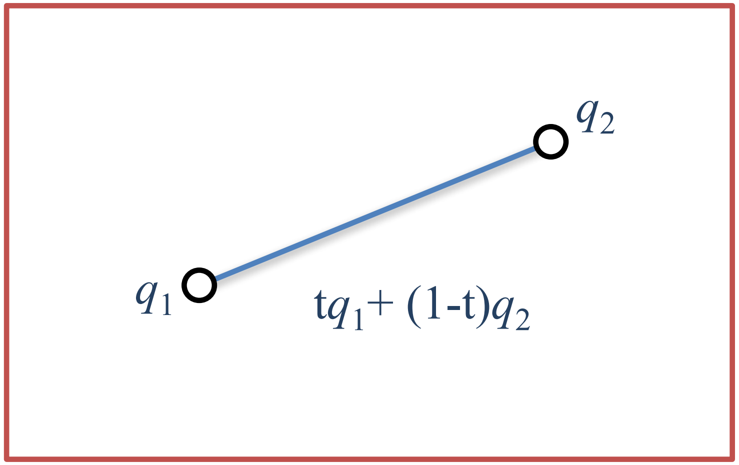


SRNF space (linear)

Geodesic paths

- Linear interpolation on SRNF space





SRNF space (linear)

Geodesic paths

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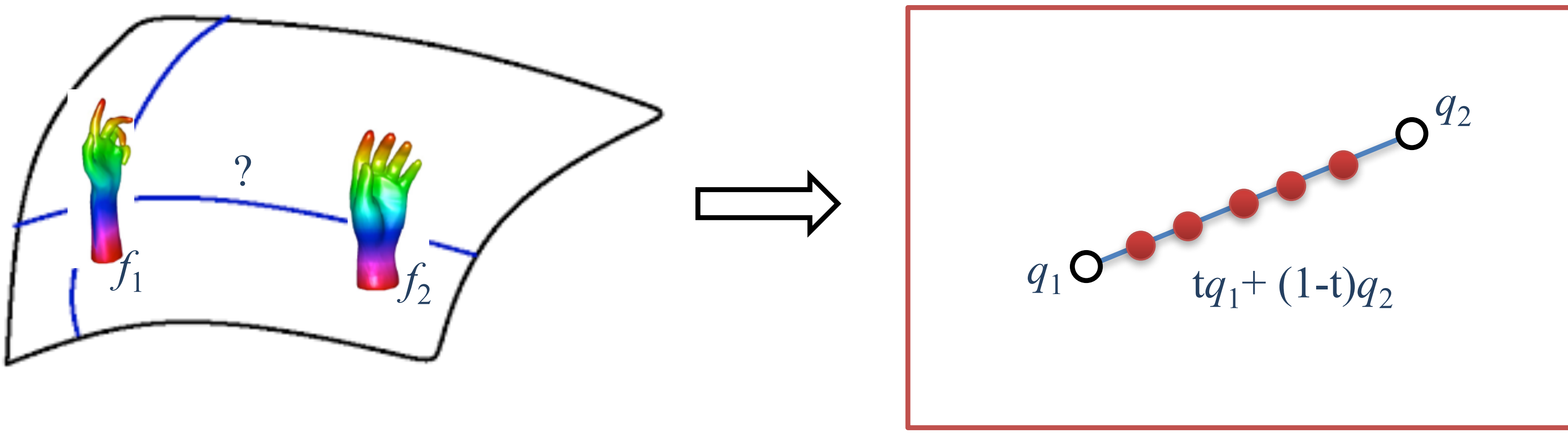
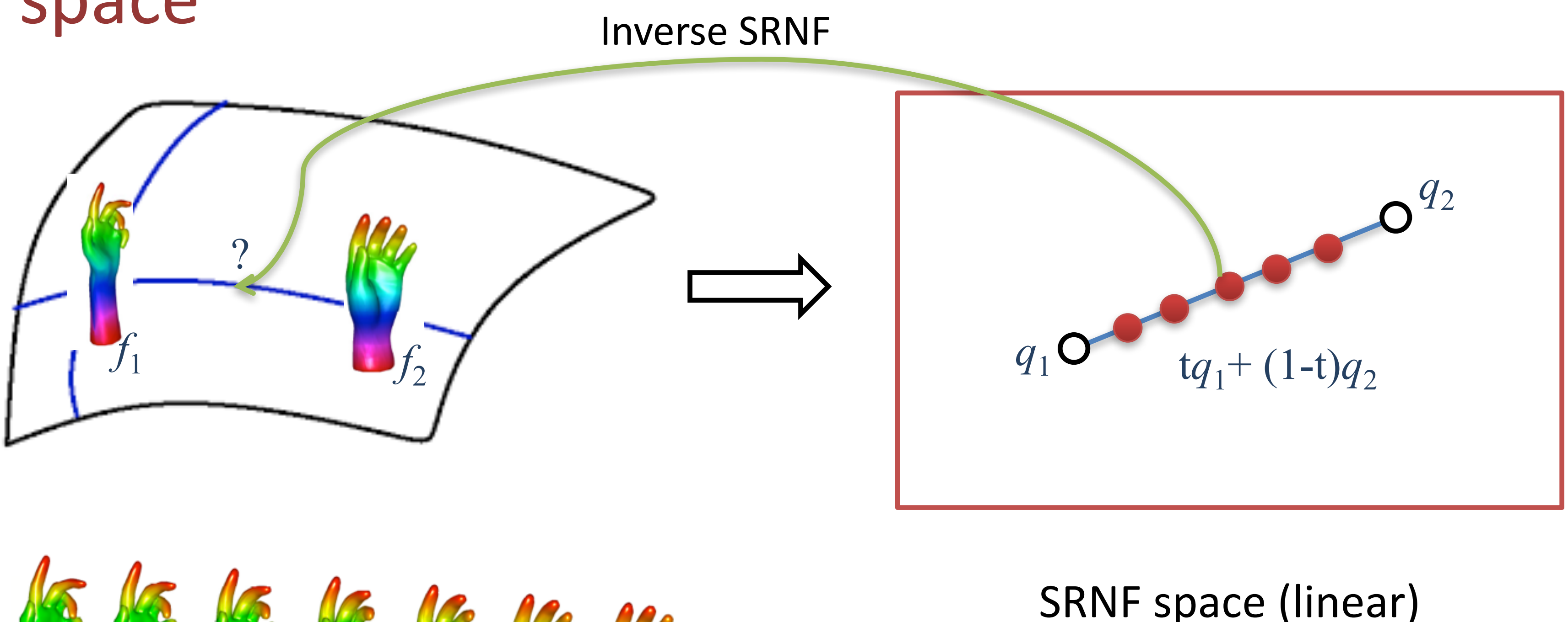
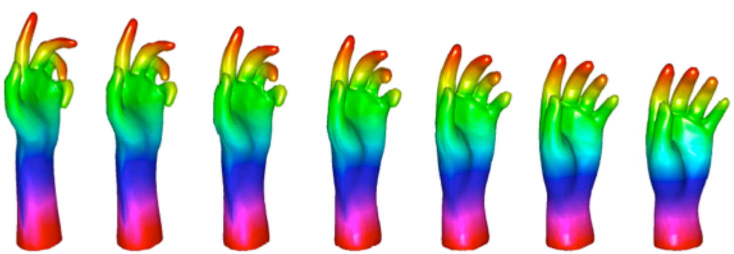


SRNF space (linear)

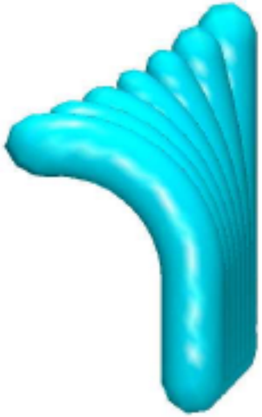
Geodesic paths

- Map lines from SRNF space back to original space



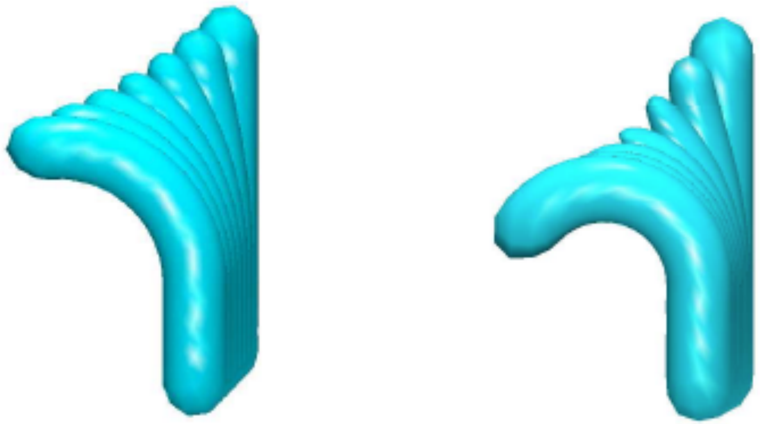


Geodesic paths



(a) Linear path $(1-t)f_1 + tf_2$

Geodesic paths



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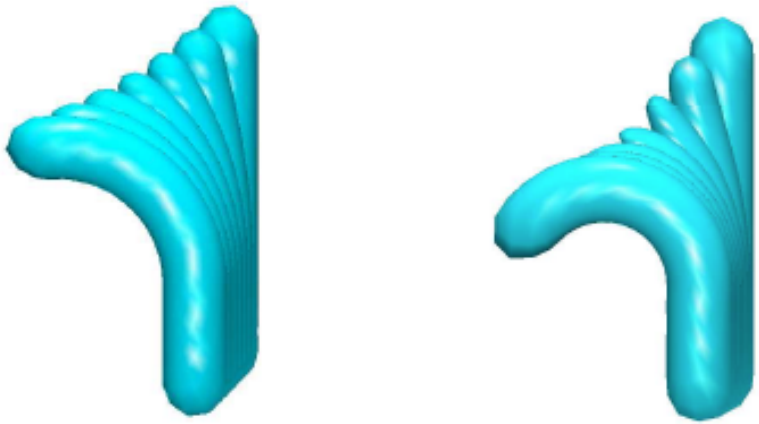
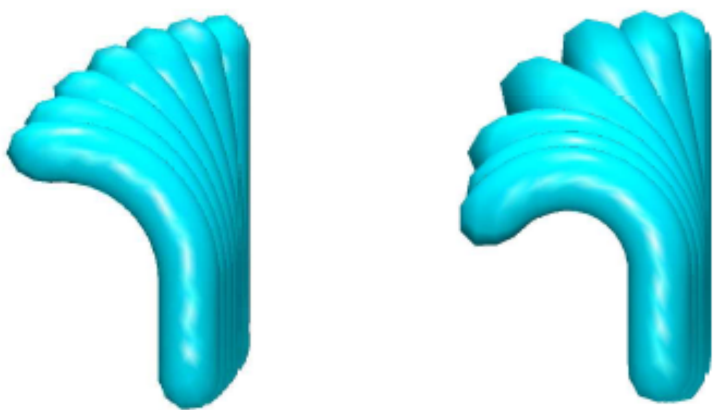
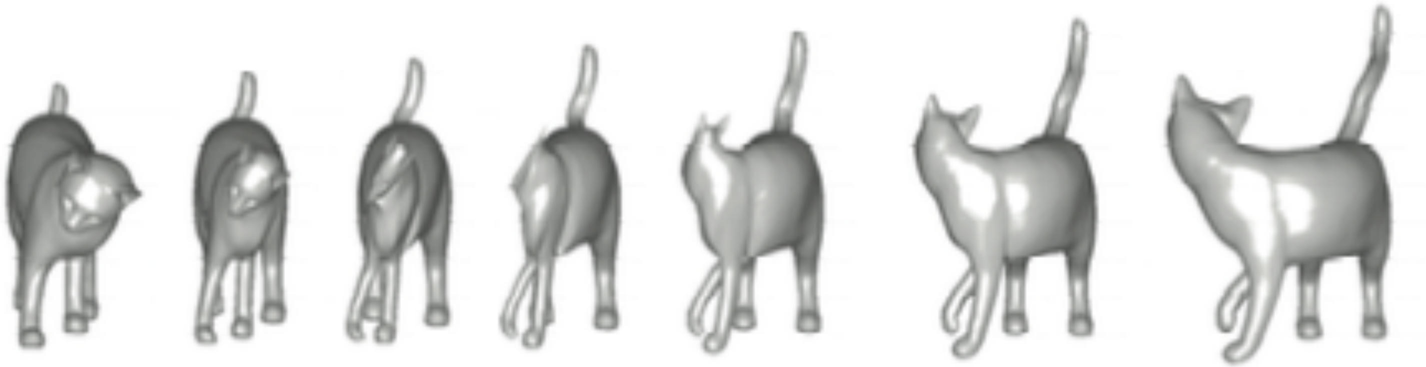


(a) Linear path $(1-t)f_1 + t f_2$



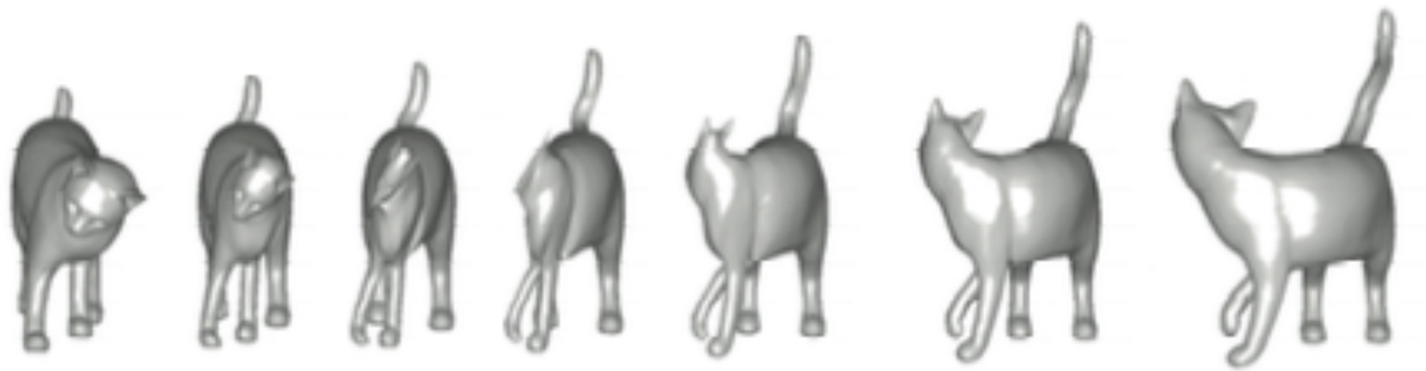
(b) Geodesic path $\alpha(t)$ by SRNF inversion

Geodesic paths



(a) Linear path $(1-t)f_1 + t f_2$
(registration computed with SRNF)

Geodesic paths



(a) Linear path $(1-t)f_1 + t f_2$
(registration computed with SRNF)

(c) Geodesic path using SRNF inversion proposed here.

Geodesic paths

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Geodesic paths

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(d) Geodesic path using SRNF inversion proposed here.

Geodesic paths

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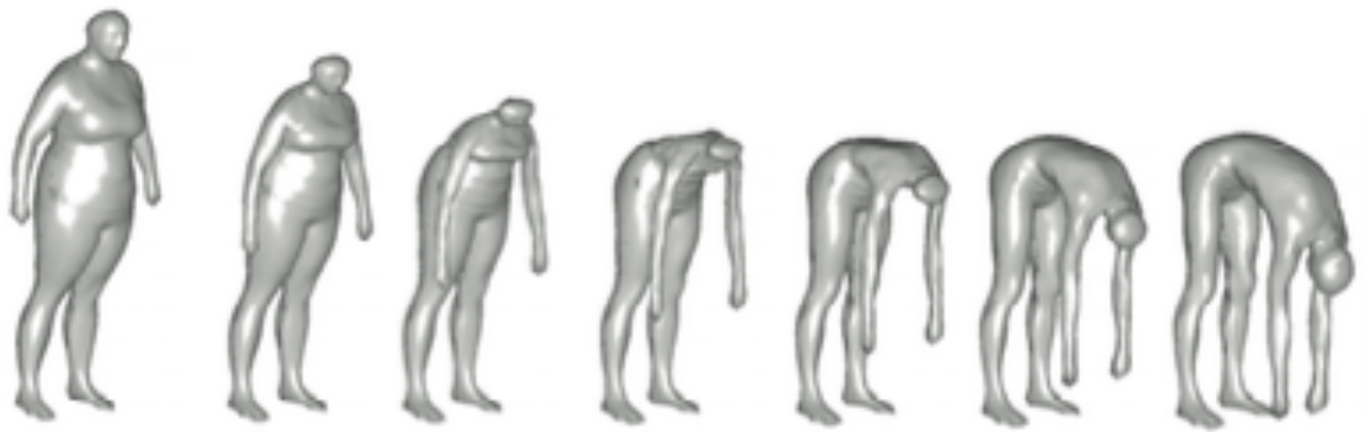
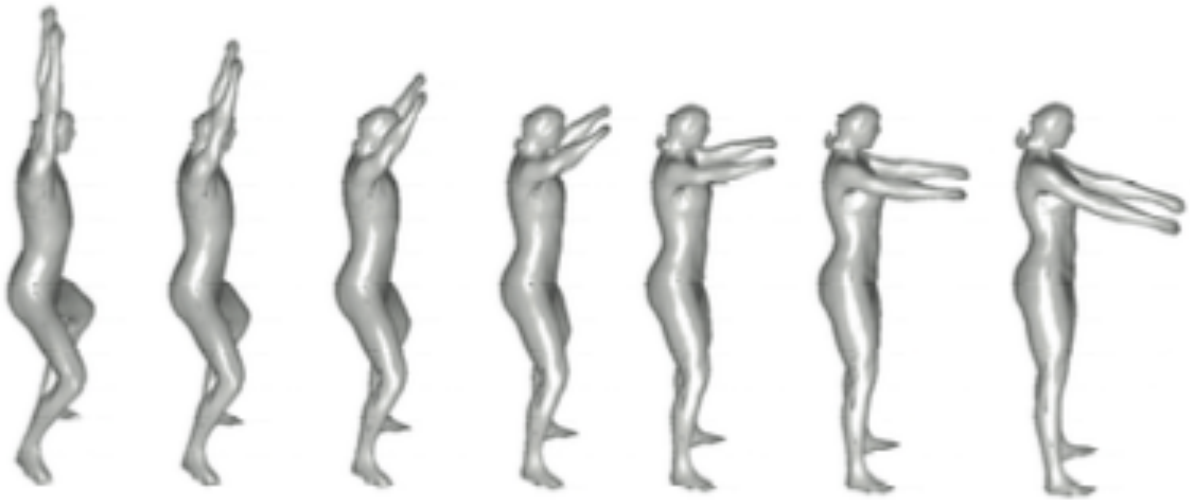
Geodesic paths



(a) Linear path with SRNF registration.

(b) Geodesic path using SRNF inversion proposed here.

Geodesic paths



(a) Linear path with SRNF registration.

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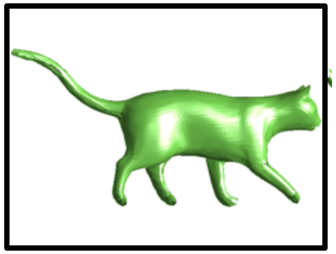
Symmetry analysis

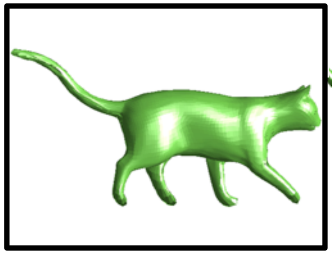
Shape symmetrization and measure of asymmetry



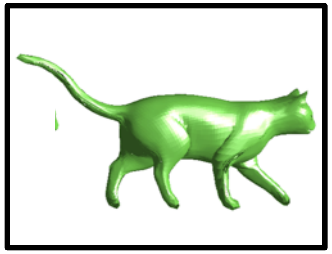
Shape f

Symmetry analysis

Shape symmetrization and measure of asymmetry



Shape f



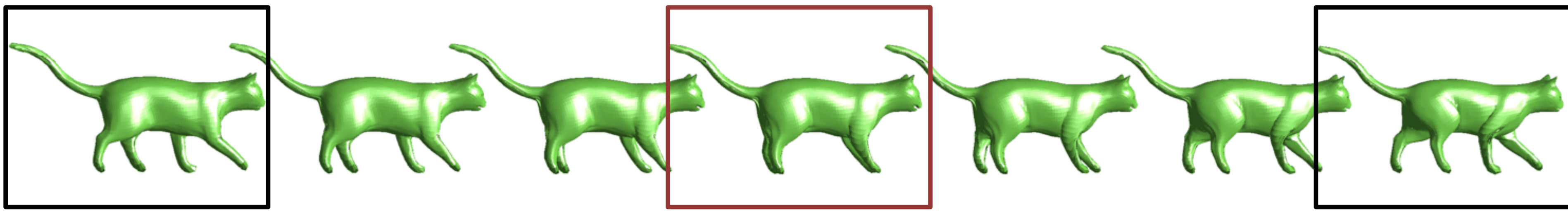
$$\tilde{f} = H(v)f$$

(Reflection of f with respect to an arbitrary plane)

$$H(v) = (I - 2\frac{vv^T}{v^T v})$$

Symmetry analysis

Shape symmetrization and measure of asymmetry



Shape f

$L(F^*) = 0.1535$

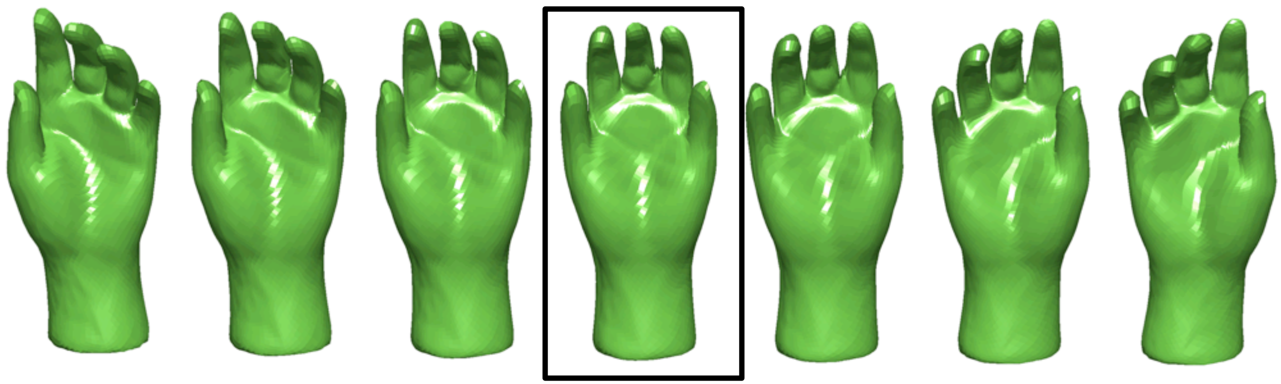
$\tilde{f} = H(v)f$

(Reflection of f with respect to an arbitrary plane)

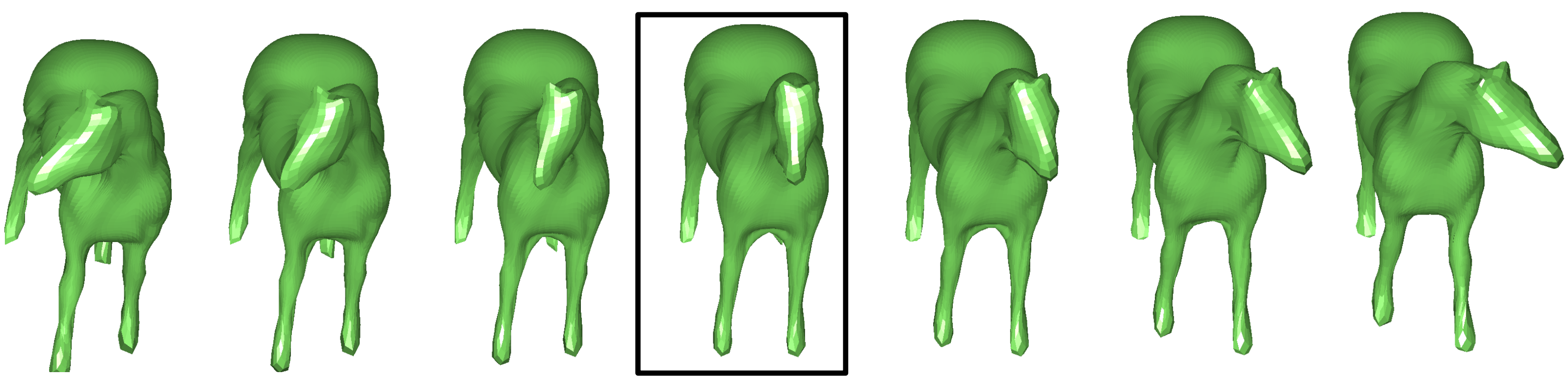
Length of the path is a measure of asymmetry

Symmetry analysis

- Shape symmetrization and measure of asymmetry

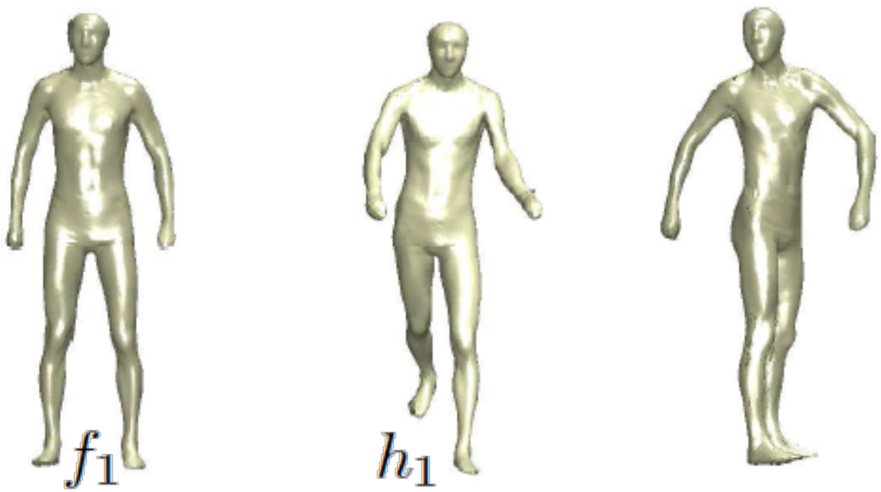


$$L(F^*) = 0.0963$$



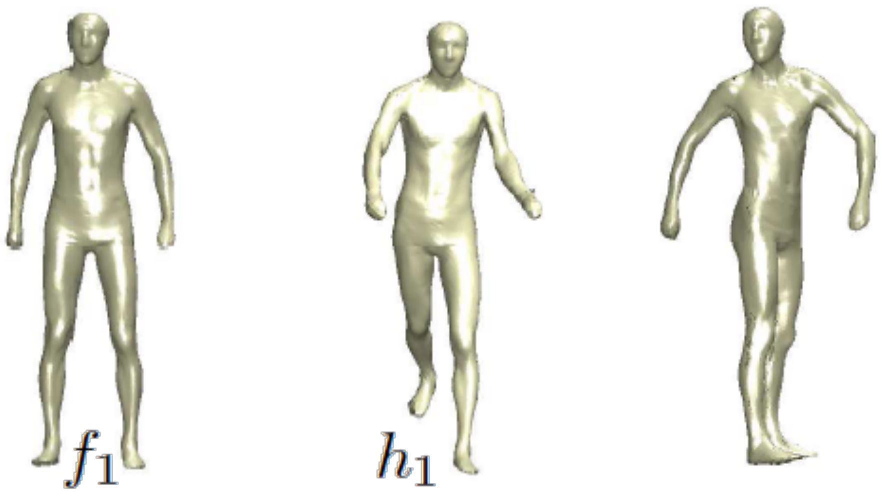
$$L(F^*) = 0.1189$$

Deformation transfer



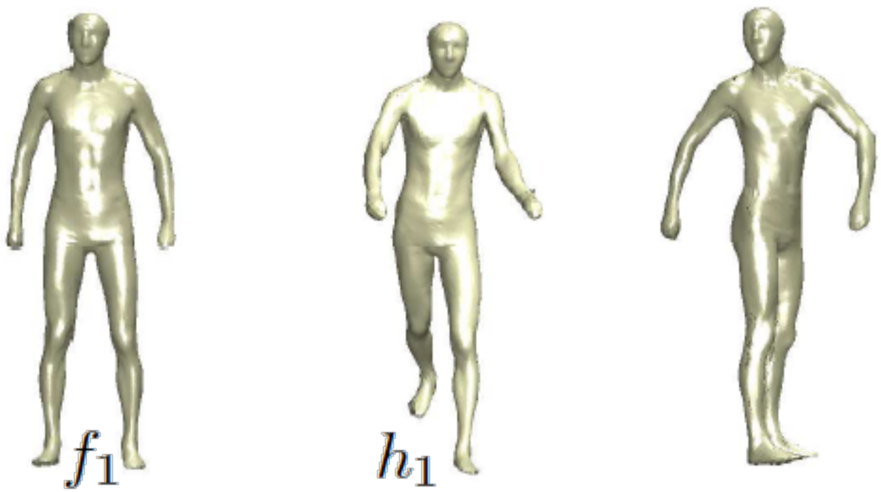
Source deformation

Deformation transfer

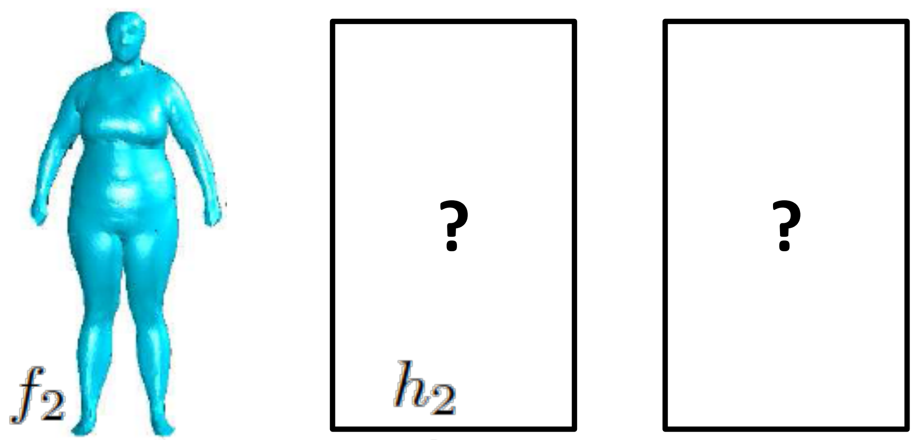


Source deformation

Deformation transfer

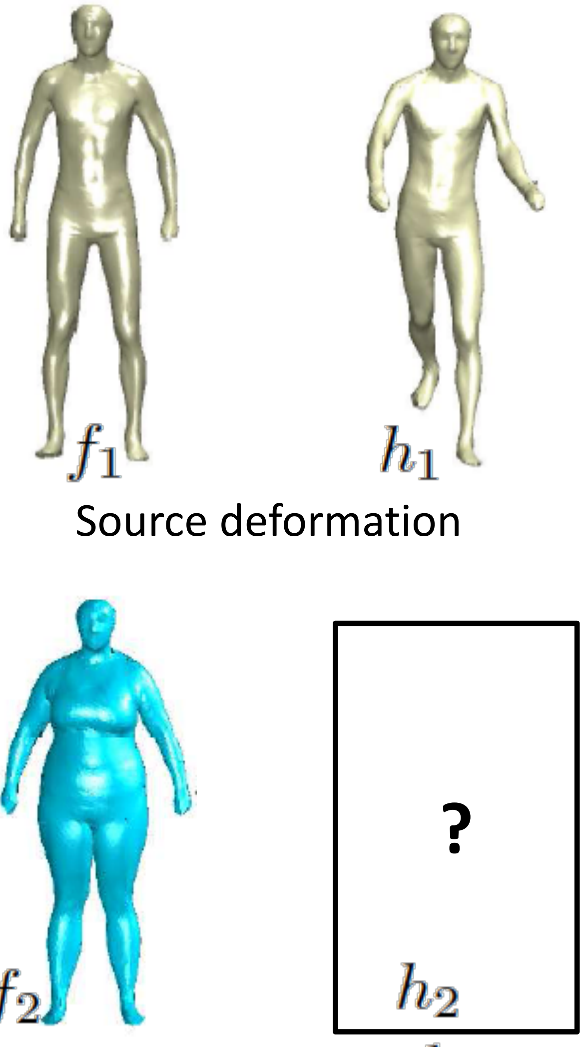


Source deformation

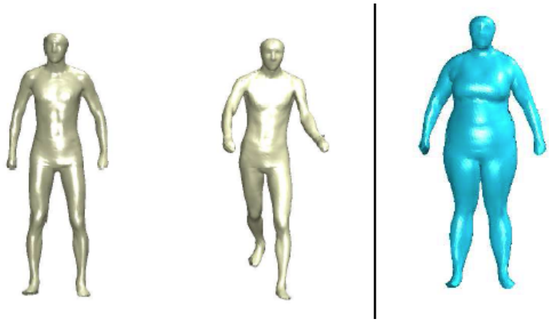


Deformation transfer

- Parallel transport in the SRNF space
  - We are given f_1, h_1, f_2, we need to find h_2
  - Compute
    - Q(f_1), Q(h_1), Q(f_2)
    - v = Q(h_1) – Q(f_1);
    - q = Q(f_2) + v
  - Invert q to obtain h_2



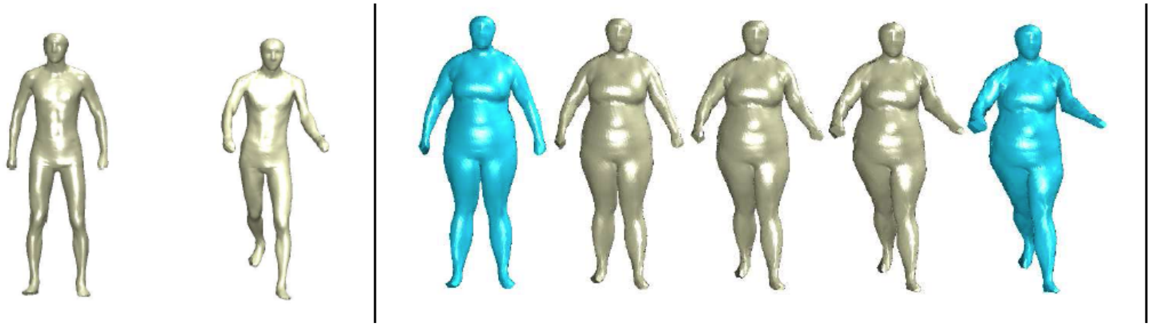
Deformation transfer results



f_1 h_1

(a) Source deformation

Deformation transfer results



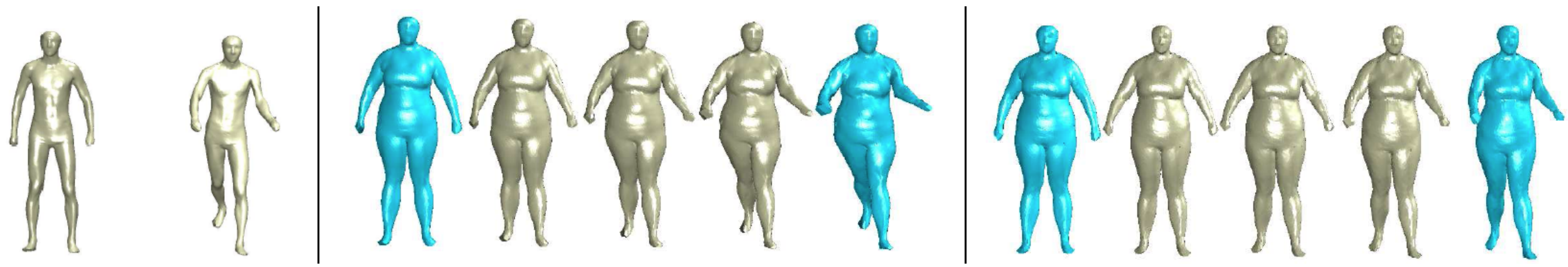
f_1 h_1

(a) Source deformation

f_2 h_2

(b) Deformation transfer by linear extrapolation, $h_2 = f_2 + \alpha(h_1 - f_1)$

Deformation transfer results



f_1 h_1

(a) Source deformation

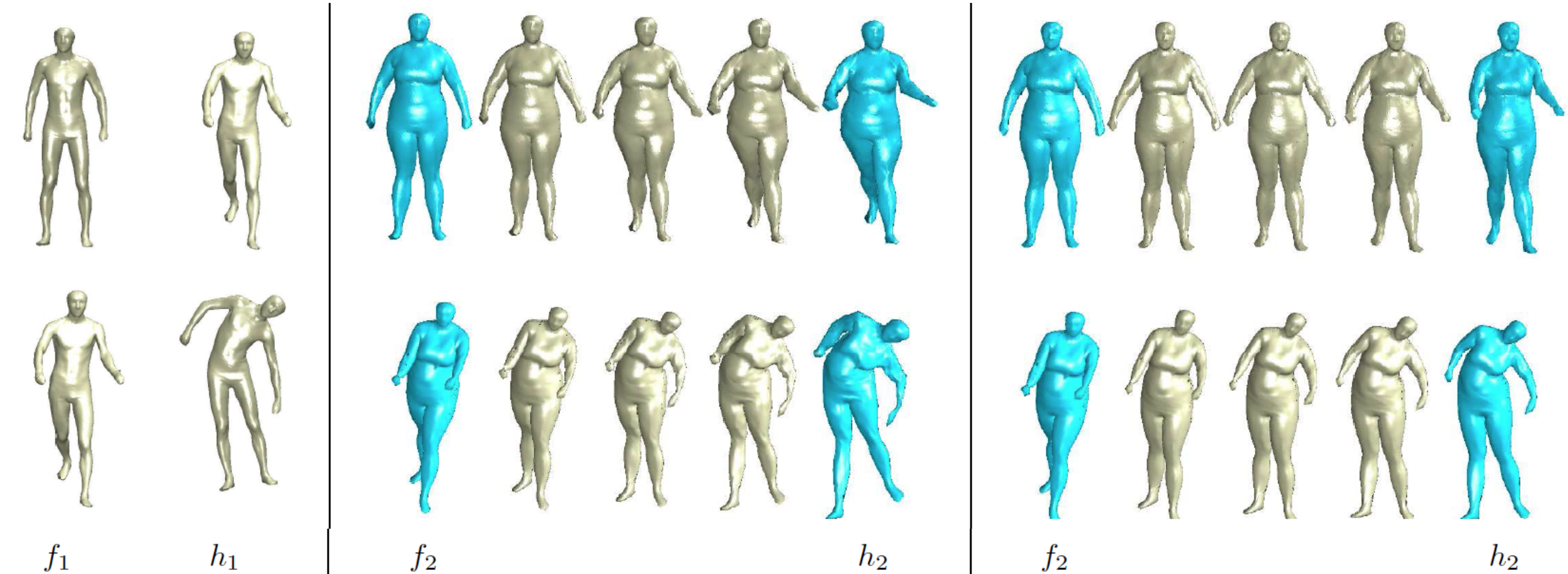
f_2 h_2

(b) Deformation transfer by linear extrapolation, $h_2 = f_2 + \alpha(h_1 - f_1)$

f_2 h_2

(c) Deformation transfer by SRNF inversion $Q(h_2) = Q(f_2) + \alpha(Q(h_1) - Q(f_1))$.

Deformation transfer results



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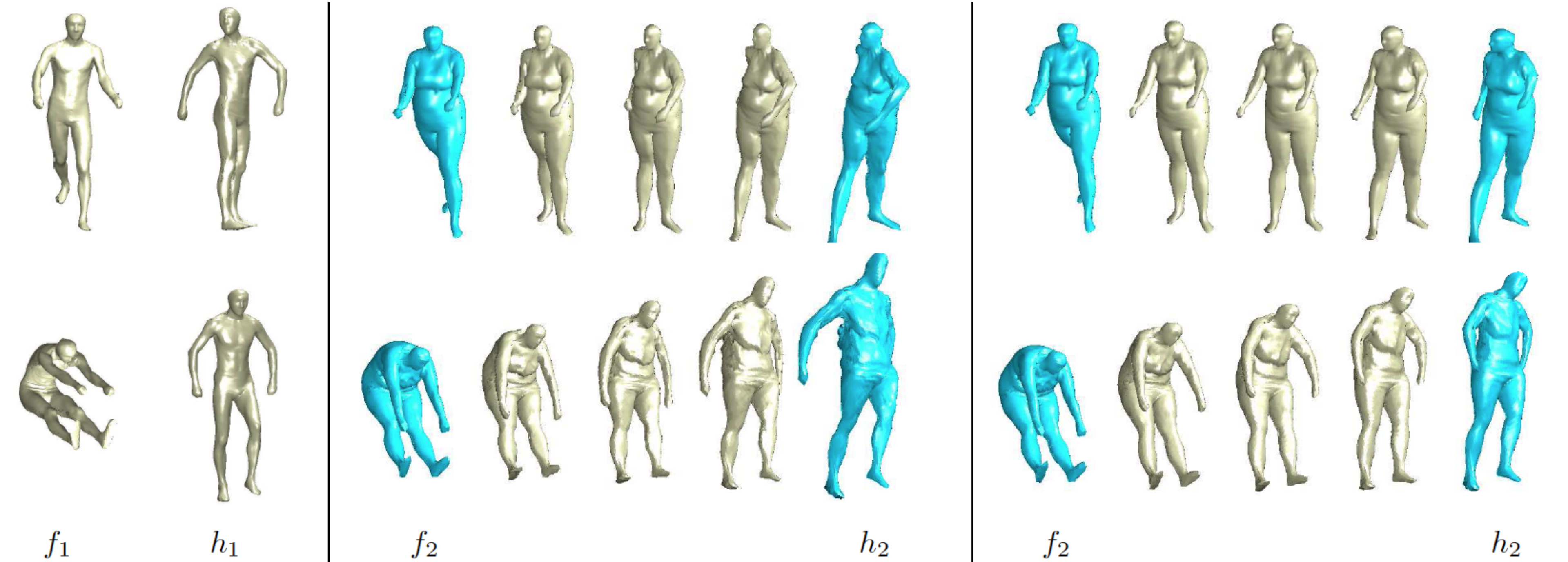
Deformation transfer results



(a) Source deformation

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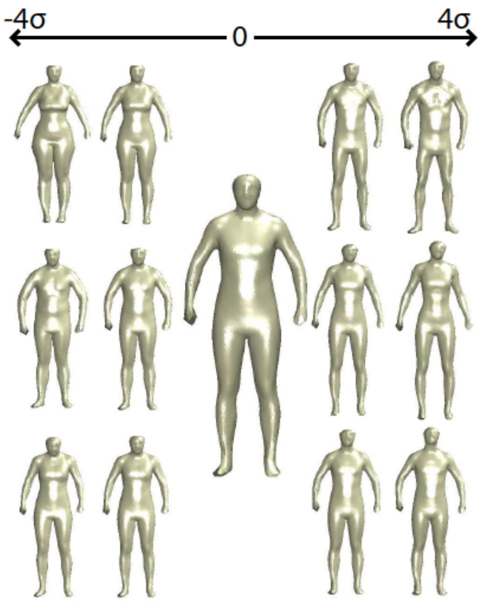
Summary statistics

- Back to our mean shape
  - Given a set of surfaces f_1, f_2, ...
  - We want to compute the mean shape and the modes of variations
- Using SRNFs
  - Compute $Q(f_1)$, $Q(f_2)$, ...
  - Use Principal Component Analysis (PCA) in the SRNF space
  - Invert the mean and principal directions back to the surface space

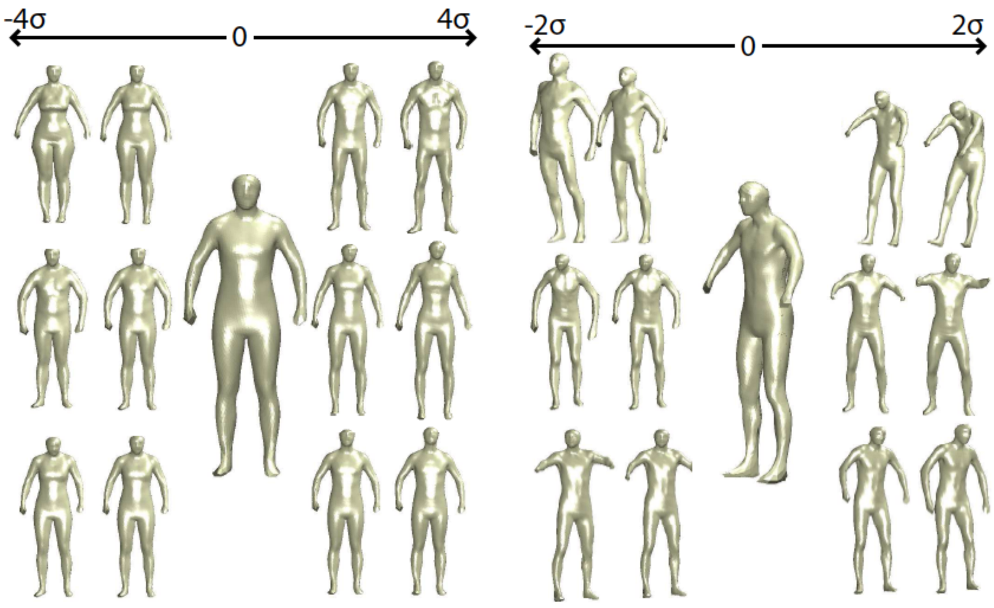
Summary statistics results



(a) Shape.

Mean and modes of variation

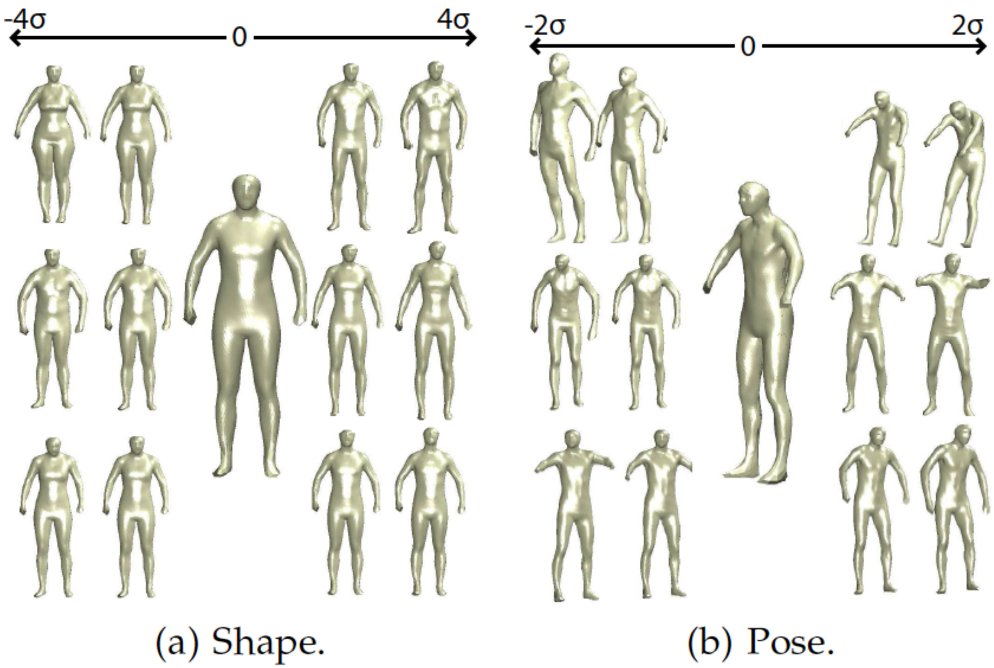
Summary statistics results

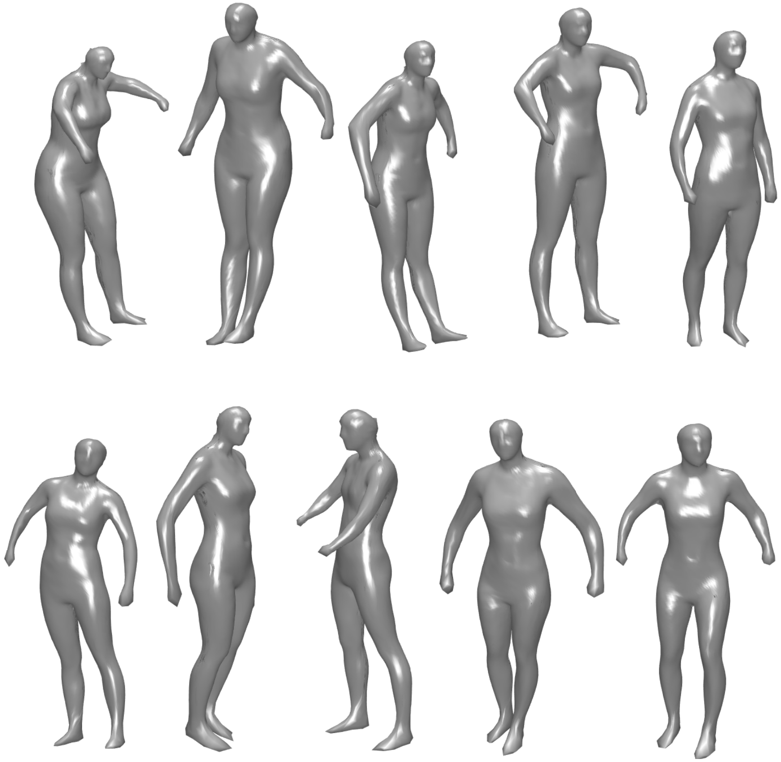


(a) Shape. (b) Pose.

Mean and modes of variation

Summary statistics results

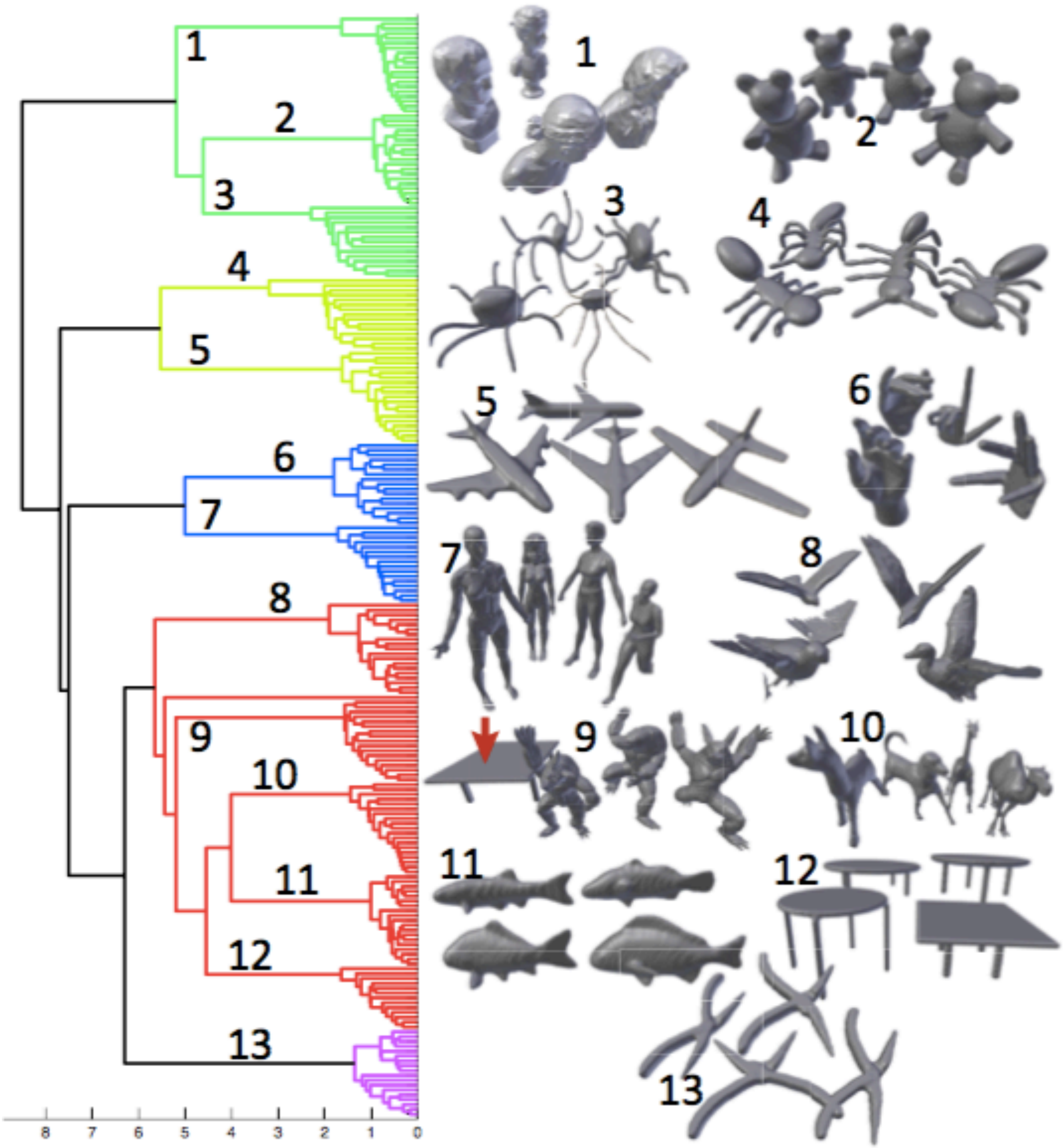




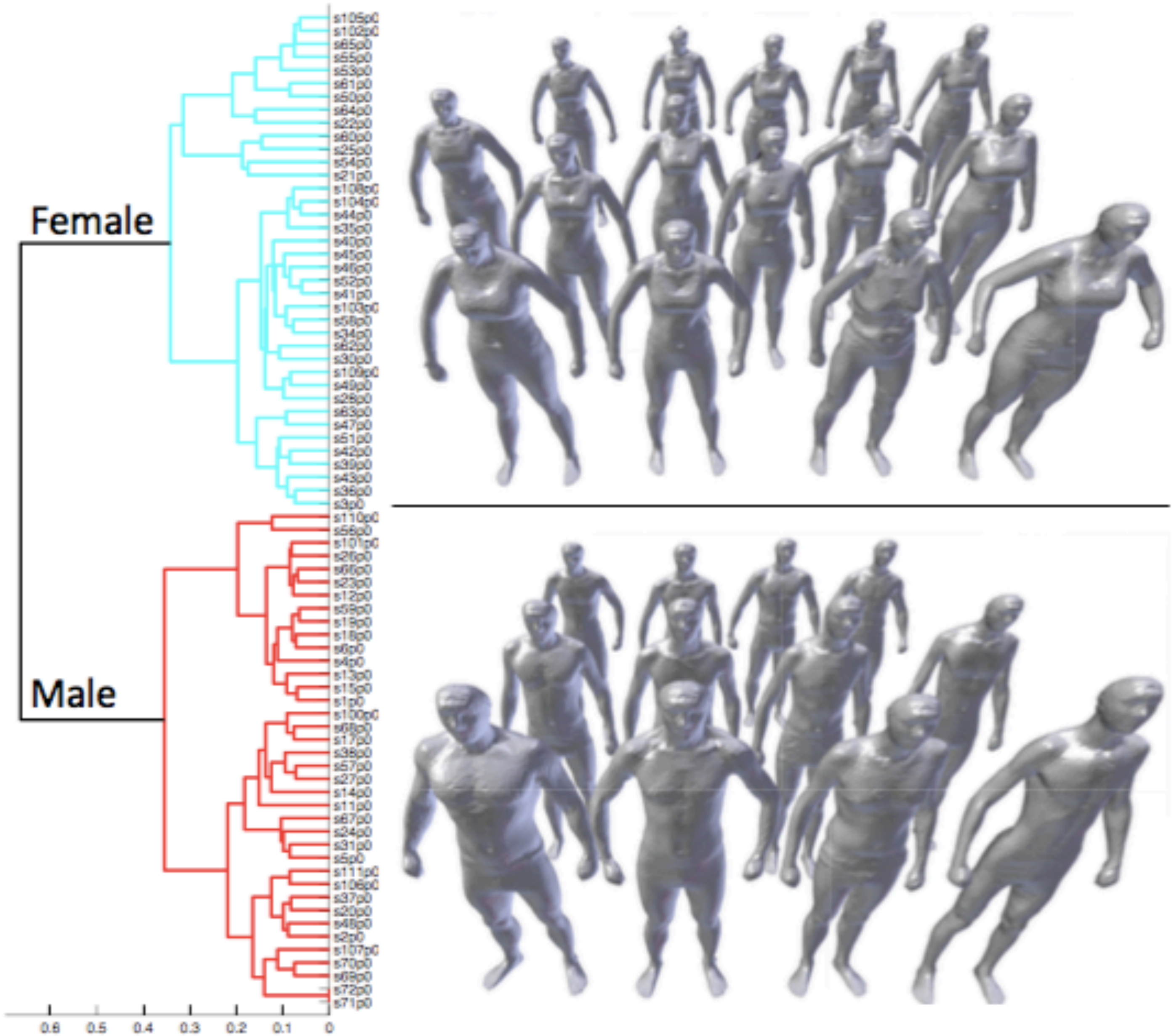
Mean and modes of variation

Random human body shapes

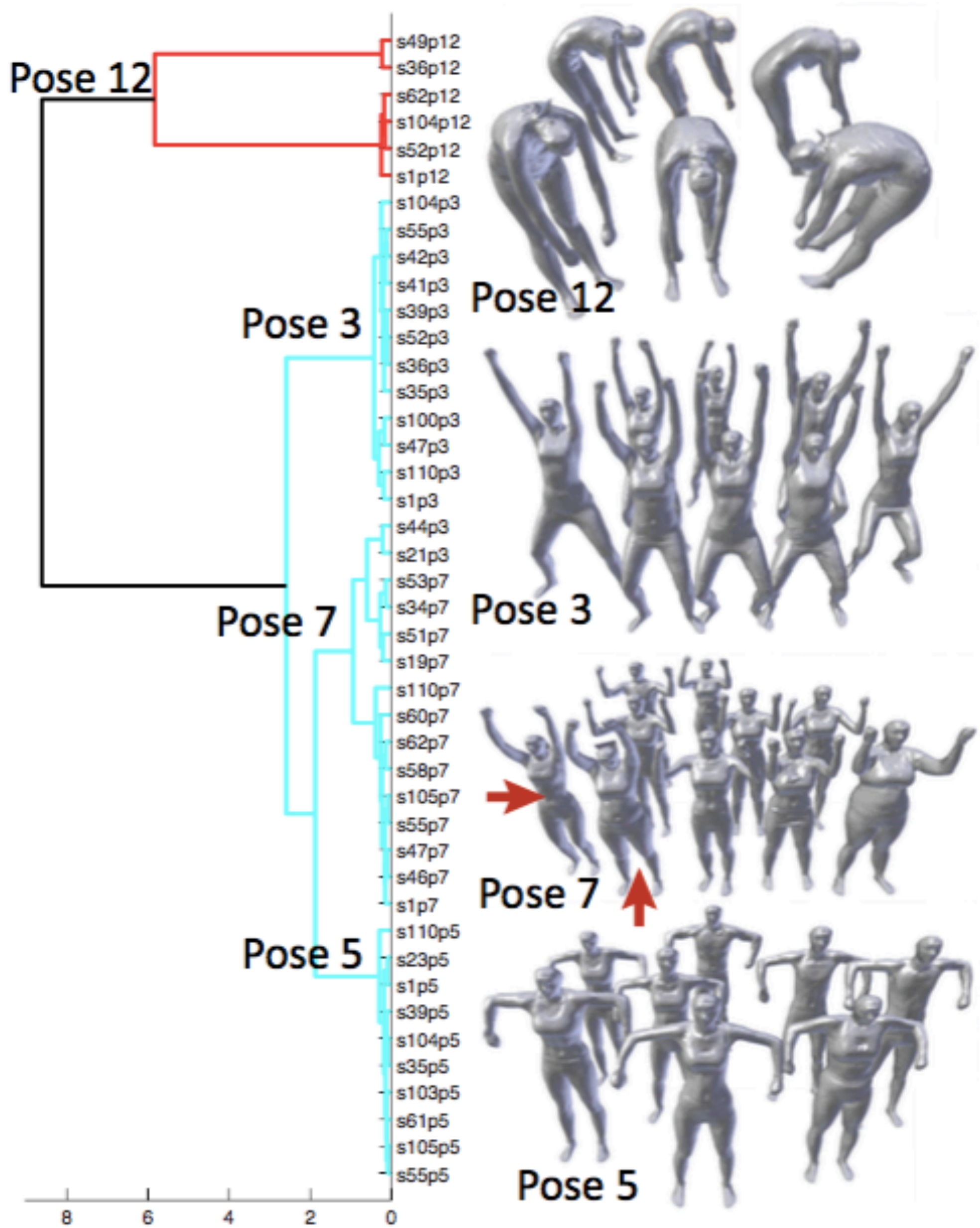
Classification of shapes



Classification of shapes

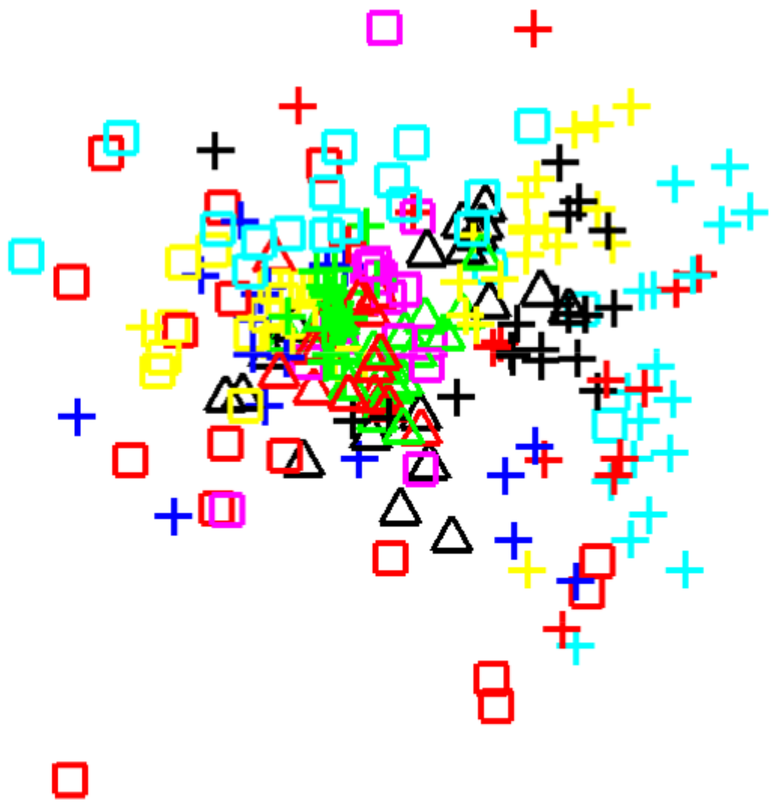


Classification of shapes



Classification of shapes – SHREC07

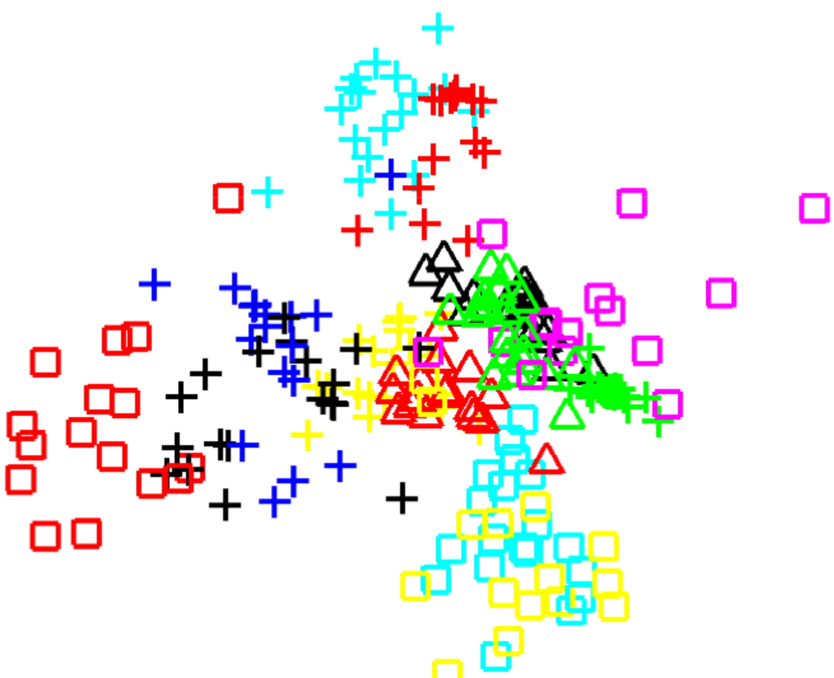
- MDS plots



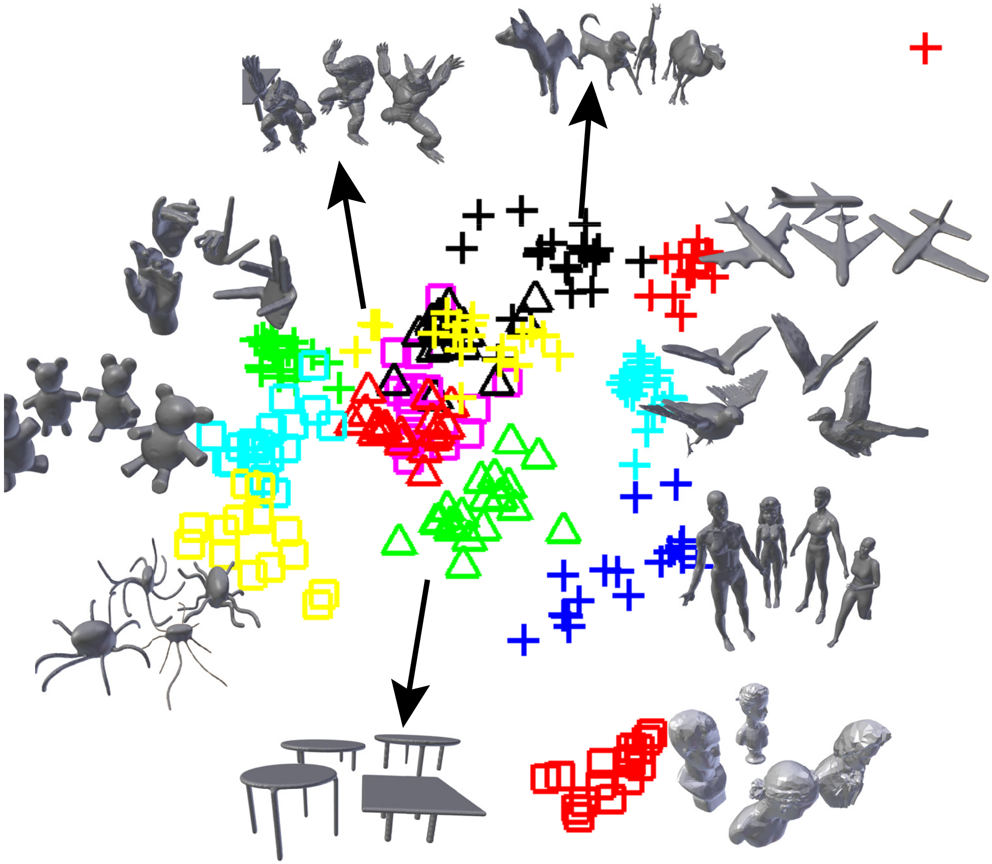
Euclidean distance between surfaces before registration

Classification of shapes – SHREC07

- MDS plots



Euclidean distance between surfaces after registration



Euclidean distance in SRNF shape space

Summary

- SRNF representation
  - Efficient registration even under large elastic deformations
  - Linearizes the shape space
    - Perform standard analysis (using vector calculus) in the space of SRNFs
    - Map the results back to the space of surfaces
- Modelling tasks become straight forward vector calculus operations
  - Deformations, deformation transfer
  - Symmetrization
  - 3D shape generation
  - Statistical classification
  - Regressions

Limitations

- The elastic registration procedure requires parameterized surfaces
  - Closed genus-0 surfaces → spherical parameterization
  - Open surfaces (e.g. human faces) → disk parameterization
  - High genus surfaces are hard to parameterize
- Do not handle topological changes
  - E.g. when a bone erodes, a hole might appear.
  - It can be a parameterization issue

Limitations

- The elastic metric
  - SRNF is a special case for alpha = ¼, beta = 1

$$d((r_1, \tilde{n}_1), (r_2, \tilde{n}_2)) = \alpha \int \frac{\partial r_1(s) \partial r_2(s)}{r(s)} ds + \beta \int \langle \partial \tilde{n}_1(s), \partial \tilde{n}_2(s) \rangle \, r(s) ds.$$

  - Ideally, we want to control the weight of each term
  - There is no such nice simplification for arbitrary alpha and beta

References

Related publications

- Hamid Laga, Qian Xie, Ian H. Jermyn, Anuj Srivastava.
  Numerical Inversion of SRNF Maps for Elastic Shape Analysis of 3D Objects.
  IEEE Transactions on Pattern Analysis and Machine Intelligence 2017
- S. Kurtek, E. Klassen, A. Srivastava and Hamid Laga.
  Landmark-Guided Elastic Shape Analysis of Spherically Parameterized Surface.
  In Eurographics 2013.
- Ian H. Jermyn, Sebastian Kurtek, Eric Klassen, and Anuj Srivastava. Elastic shape matching of parameterized surfaces using square root normal fields. ECCV2012.
- S Kurtek, E Klassen, JC Gore, Z Ding, A Srivastava. Elastic geodesic paths in shape space of parameterized surfaces. IEEE Transactions on Pattern Analysis and Machine Intelligence 2012.
- S Kurtek, E Klassen, Z Ding, SW Jacobson, JL Jacobson, MJ Avison. Parameterization-invariant shape comparisons of anatomical surfaces. IEEE Transactions on Medical Imaging 2011.

How about objects with structural variability?