A Logic for Global and Local Announcements

F. Belardinelli

Laboratoire IBISC, UEVE & IRIT Toulouse

joint work with H. van Ditmarsch and W. van der Hoek @TARK’17

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Outline

1. **Background:** logics for (public, semi-private, private) announcements \([vDHvdHK15]\)
   
   In PAL announcements are
   
   - **public:** all agents listen to (and are aware of!) the announcement
   - **global:** how the new information is processed depends on the model (i.e., public announcements are model transformers)

2. **Goal:** to generalise PAL by weakening **publicity** and **globality**
   
   - **privacy:** announcements to any subset \(A \subseteq Ag\) of agents
   - **locality:** announcements are **pointed model** transformers

3. **Dynamic Epistemic Logic:** action models allow private announcements, but
   
   - updated indistinguishability relations are not necessarily equivalences
   - updated models might be strictly larger . . .
   - . . . several problems are undecidable

4. **GLAL:** an extension of PAL supporting both **private** and **local** announcements
   
   - updated indistinguishability relations are equivalences
   - updated models are normally “smaller” . . .
   - . . . the model checking and satisfaction problems are decidable
Let $Ag$ be a set of agents and $AP$ a set of propositional atoms.

**Definition (GLAL)**

Formulas $\phi$ in $L_{glal}$ are defined by the following BNF:

$$\psi ::= p | \neg \psi | \psi \land \psi | K_a \psi | C_A \psi | [\psi]^+_A \psi | [\psi]^-_A \psi$$

- $[\psi]^+_A \phi ::= \text{after globally announcing $\psi$ to the agents in $A$, $\phi$ is true}$
- $[\psi]^-_A \phi ::= \text{after locally announcing $\psi$ to the agents in $A$, $\phi$ is true}$

$$L_{pl} \subseteq L_{el} \subseteq L_{pal^+} \subseteq L_{glal}$$
Formulas in GLAL are interpreted on (multi-modal) Kripke models.

**Definition (Frame)**

A **frame** is a tuple $\mathcal{F} = \langle W, \{R_a\}_{a \in Ag} \rangle$ where

- $W$ is a set of **possible worlds**
- for every agent $a \in Ag$, $R_a \subseteq 2^{W \times W}$ is an **equivalence relation** on $W$.

A **model** is a pair $\mathcal{M} = \langle \mathcal{F}, V \rangle$ where $V : AP \rightarrow 2^W$ is an assignment to atoms.

- $R^C_A = (\bigcup_{a \in A} R_a)^*$ is the reflexive and transitive closure of $\bigcup_{a \in A} R_a$
- $R(w) = \{ w' \in W \mid R(w, w') \}$ is the $R$-equivalence class of $w \in W$
Satisfaction & Refinements

The satisfaction set $[[\varphi]]M \subseteq W$ is defined as

$[[p]]M = V(p)$

$[[\neg \psi]]M = W \setminus [[\psi]]M$

$[[\psi \land \psi']]M = [[\psi]]M \cap [[\psi']]M$

$[[C_A \psi]]M = \{ w \in W \mid \text{for all } w' \in R^C_A(w), w' \in [[\psi]]M \}$

$[[\psi]_A^\land \psi']M = \{ w \in W \mid \text{if } w \in [[\psi]]M \text{ then } w \in [[\psi']]M^{-(w,\psi,A)} \}$

$[[\psi]_A^\lor \psi']M = \{ w \in W \mid \text{if } w \in [[\psi]]M \text{ then } w \in [[\psi']]M^{+(w,\psi,A)} \}$

where refinements $M^{-(w,\psi,A)} = \langle W^-, \{ R^-_a \}_{a \in Ag}, V^- \rangle$ and $M^{+(w,\psi,A)} = \langle W^+, \{ R^+_a \}_{a \in Ag}, V^+ \rangle$ have

- $W^- = W^+ = W$ and $V^- = V^+ = V$
- for every agent $b \notin A$, $R^-_b = R^+_b = R_b$; while for $a \in A,$

$$R^-_a(v) = \begin{cases} R_a(v) \cap [[\psi]]M & \text{if } v \in R_a(w) \cap [[\psi]]M \\ R_a(v) \cap [[\neg \psi]]M & \text{if } v \in R_a(w) \cap [[\neg \psi]]M \\ R_a(v) & \text{otherwise} \end{cases}$$

$$R^+_a(v) = \begin{cases} R_a(v) \cap [[\psi]]M & \text{if } v \in R^C_a(w) \cap [[\psi]]M \\ R_a(v) \cap [[\neg \psi]]M & \text{if } v \in R^C_a(w) \cap [[\neg \psi]]M \\ R_a(v) & \text{otherwise} \end{cases}$$

Remark

- for every agent $a \in Ag$, $R^-_a$ and $R^+_a$ are equivalence relations
- $[\psi]^+_A$ and $[\psi]^-_A$ are interpreted as local (pointed model) transformers
- the difference between global and local announcements collapse whenever $A$ is a singleton.
Example:
Example: the Muddy Children Puzzle
Example: the Muddy Children Puzzle

The model \( \mathcal{M} \) for 3 children (red, blue, and green), where no child knows whether she is muddy, can be represented as follows:
Example: the Muddy Children Puzzle

- Suppose that only red is muddy, i.e., the actual world is (1, 0, 0)
- then, the father **locally** announces to red and blue that at least one child is muddy: 
  \[ \alpha := m_r \lor m_b \lor m_g \]
- the updated model \( \mathcal{M}_{(100, \alpha, rb)}^- \) is as follows:

  ![Diagram]

- only the indistinguishability relation for red is updated
- now red and blue both know that at least one child is muddy: \( (\mathcal{M}, 100) \models [\alpha]^-_{rb} E_{rb} \alpha \)
- the father’s announcement does not make \( \alpha \) common knowledge: \( (\mathcal{M}, 100) \not\models [\alpha]^-_{rb} C_{rb} \alpha \)
- In general, for every world \( w \neq 000 \), \( (\mathcal{M}, w) \not\models [\alpha]^-_{rb} C_{rb} \alpha \)
Example: the Muddy Children Puzzle

- Suppose that the father **globally** announces to red and blue that at least one child is muddy
- the updated model $\mathcal{M}^+_{(100,\alpha,rb)}$ is as follows:

```
(1,1,1)
/  \  /
/   \ /
/     /
(1,0,1)  (0,1,1)
```

- now the indistinguishability relations for both red and blue are updated and . . .
  . . . they acquire common knowledge that at least one child is muddy: $(\mathcal{M}, 100) \vDash [\alpha]_{rb}^+ C_{rb} \alpha$
- but the father’s announcement is not enough to make $\alpha$ common knowledge amongst all children: $(\mathcal{M}, 100) \not\vDash [\alpha]_{rb}^+ C_{rgb} \alpha$
Example: Communication Scenario

Consider communication between sender $s$ and receiver $r$ over a reliable channel that is listened to by eavesdropper $e$:

$$
\begin{array}{c}
\text{\textcolor{red}{$w_1$}} \quad 0 \quad r, e \quad 1 \quad \text{\textcolor{red}{$w_2$}} \\
\end{array}
$$

After $s$ has communicated to $r$ the value of the bit, we obtain the updated model $N(w_1, \text{\textcolor{red}{bit}} = 0, r)$:

Hence, receiver $r$ learns the value of the bit:

$$
N(w_1, \text{\textcolor{red}{bit}} = 0, r) / K_r(\text{\textcolor{red}{bit}} = 0)
$$

On the other hand, eavesdropper $e$ learns that $r$ knows it:

$$
N(w_1, \text{\textcolor{red}{bit}} = 0, e) / K_e K_w r(\text{\textcolor{red}{bit}} = 0)
$$
Example: Communication Scenario

Consider communication between sender $s$ and receiver $r$ over a reliable channel that is listened to by eavesdropper $e$:

$$w_1 \ x \ 0 \ x \ r, e \ x \ 1 \ x \ w_2$$

After $s$ has communicated to $r$ the value of the bit, we obtain the updated model $N(w_1, bit=0, r)$:

$$w_1 \ x \ 0 \ x \ e \ x \ 1 \ x \ w_2$$

Hence, receiver $r$ learns the value of the bit: $(N, w_1) \models [bit = 0]_r K_r (bit = 0)$

On the other hand, eavesdropper $e$ learns that $r$ knows it: $(N, w_1) \models [bit = 0]_r K_e K_w_r (bit = 0)$
Example: Communication Scenario

Compare model \( N \) above with the following **bisimilar** model \( N' \),

\[
\begin{array}{c}
v'_1 & 0 & r, e & 1 & v'_2 \\
| & s, e & \downarrow & | & s, e \\
w'_1 & 0 & r, e & 1 & w'_2
\end{array}
\]

However, after communicating the value of the bit, the updated model \( N'(w'_1, \text{bit} = 0, r) \) is not bisimilar to \( N(w_1, \text{bit} = 0, r) \):

In particular, in \( w'_1 \) the eavesdropper does not learn that \( r \) knows the value of the bit: \( (N', w'_1, \text{bit} = 0) /\not\sim (N, w_1, \text{bit} = 0, r) \).
Example: Communication Scenario

Compare model $N$ above with the following bisimilar model $N'$,

However, after communicating to $r$ the value of the bit, the updated model $N'_{(w_1',bit=0,r)}$ is not bisimilar to $N_{(w_1,bit=0,r)}$:

In particular, in $w_1'$ eavesdropper $e$ does not learn that $r$ knows the value of the bit: $(N',w_1') \not\equiv [bit = 0]_r K_e K_w r (bit = 0)$.

$\Rightarrow$ GLAL is not preserved under standard modal bisimulations.
Comparison with PAL

GLAL is at least as expressive as PAL:

**Proposition**

For all formulas $\phi, \psi$ in PAL, $(M, w) \models [\phi] \psi$ iff $(M, w) \models [\phi]_{Ag} \psi$.

By this result we can define a truth-preserving embedding $\tau$ from PAL to GLAL.

**Proposition**

For all formulas $\phi$ in PAL, $(M, w) \models \phi$ iff $(M, w) \models \tau(\phi)$.

Actually, by the example above,

**Theorem**

GLAL is strictly more expressive than PAL, and therefore than epistemic logic.
Comparison with Attentive Announcements

- **Attention-based Announcements** [BDH⁺16]: agents process the new information only if they are paying attention.
- whether they pay attention is handled by a designated set of atoms.
- close relationship with GLAL: in \((\mathcal{N}', w'_1)\) although \(r\) processes the new information, agent \(s\) is uncertain about this fact.
- consider adding an ‘attention atom’ \(h_r\) for receiver \(r\) such that \(h_r\) is true in \(w'_1\) and \(w'_2\) but false in \(v'_1\) and \(v'_2\).
- then, announcing \(bit = 0\) to \(r\) in \((\mathcal{N}', w'_1)\) corresponds to the attention-based announcement wherein sender \(s\) is uncertain as to whether \(r\) is paying attention.

**Differences:**

- [BDH⁺16] models truly private announcements [GG97] (equivalence relations are not preserved), whereas our proposal considers semi-private announcements that do preserve equivalence relations.
- Our announcements are not necessarily public.
Comparison with Semi-Private Announcements

- **Semi-Private Announcements** [GG97, vD00, vdHP06, BvDM08]: after announcing semi-prvately $\phi$ to coalition $A$, all agents in $A$ know $\phi$, and the agents in $Ag \setminus A$ know that all agents in $A$ know whether $\phi$.

- In GLAL agents in $Ag \setminus A$ do not necessarily know that all agents in $A$ know whether $\phi$.

- Semi-private announcements can be modeled by refinement $\mathcal{M}^{sp}_{(w, \psi, A)}$ according to which $W^{sp} = W$, $V^{sp} = V$, and for $a \in A$,

$$R^{sp}_a(v) = \begin{cases} R_a(v) \cap [[\psi]]_\mathcal{M} & \text{if } v \in R^{C}_{Ag}(w) \cap [[\psi]]_\mathcal{M} \\ R_a(v) \cap [[\neg \psi]]_\mathcal{M} & \text{if } v \in R^{C}_{Ag}(w) \cap [[\neg \psi]]_\mathcal{M} \\ R_a(v) & \text{otherwise} \end{cases}$$

- The two frameworks are not directly comparable.
Validities

No complete axiomatisation, but some interesting validities.

- Truthfully announcing a propositional formula $\phi \in L_{pl}$ entails the knowledge thereof:
  \[
  \models [\phi]_A E_A \phi
  \]
  \[
  \models [\phi]_A^+ C_A \phi
  \]

- Differently from PAL, announcements in GLAL cannot be rewritten as simpler formulas. Nonetheless, the following are validities in GLAL:
  \[
  [\phi]_A^p \leftrightarrow \phi \to p
  \]
  \[
  [\phi]_A \neg \psi \leftrightarrow \phi \to \neg [\phi]_A \psi
  \]
  \[
  [\phi]_A (\psi \land \psi') \leftrightarrow [\phi]_A^\neg \psi \land [\phi]_A^\neg \psi'
  \]

- Further, epistemic operators and nested announcements commute with announcement operators if they refer to the same coalition (but not in general):
  \[
  [\phi]_A^E E_A \psi \leftrightarrow \phi \to E_A [\phi]_A^E \psi
  \]
  \[
  [\phi]_A [\phi']_A^E \psi \leftrightarrow [\phi \land [\phi]_A \phi']_A^E \psi
  \]
  \[
  [\phi]_A [\phi']_A^+ \psi \leftrightarrow [\phi \land [\phi]_A \phi']_A^+ \psi
  \]

- Operators $[\phi]_A^+$ and $[\phi]_A^-$ are “normal” modalities.

None of schemes T, S4 and B hold.
A New Notion of Bisimulation

We remarked that GLAL is not preserved under modal bisimulation.

- define $R_A(w, v)$ as: $R_a(w, v)$ iff $a \in A$.

**Definition ($\pm$-Simulation)**

Given models $\mathcal{M}$ and $\mathcal{M}'$, a $\pm$-simulation is a relation $S \subseteq W \times W'$ such that $S(w, w')$ implies

- **Atoms** $w \in V(p)$ iff $w' \in V'(p)$, for every $p \in AP$
- **Forth** for every $A \subseteq Ag$ and $v \in W$, if $R_A(w, v)$ then for some $v' \in W'$, $R'_A(w', v')$ and $S(v, v')$
- **Reach** for every $v, v' \in W$, $a \in Ag$, if $S(v, v')$ then $R_a(w, v)$ iff $R'_a(w', v')$

**Theorem**

*If states $s$ and $s'$ are bisimilar, then for every formula $\psi$ in GLAL, $(\mathcal{M}, s) \models \psi$ iff $(\mathcal{M}', s') \models \psi$.*
Model Checking and Satisfiability

Definition (Model Checking and Satisfiability)

- **Model Checking Problem**: given a finite pointed model \((M, w)\), and formula \(\phi\) in GLAL, determine whether \((M, w) \vDash \phi\).

- **Satisfiability Problem**: given a formula \(\phi\) in GLAL, determine whether \((M, w) \vDash \phi\) for some pointed model \((M, w)\).

Theorem

*The model checking problem for GLAL is PTIME-complete.*

Model refinements can be computed in polynomial time.

Theorem

*The satisfiability problem for GLAL is decidable.*

Decision procedure inspired by tableaux for epistemic logic.
Conclusions

Contributions:

- GLAL: a logic for global and local announcements
- strictly more expressive than PAL
- alternative to action models to represent private announcements
- however, not preserved under standard modal bisimulation
- but we have a novel, truth-preserving notion of bisimulation
- the model checking problem is no harder than for epistemic logic
- the satisfiability problem is decidable.

Future Work:

- axiomatisation
- closer comparison with DEL
- more elaborate form of communication (asynchronous, FIFO, LIFO, etc.)
- real-life scenarios and applications

- to be held in Evry (UEVE), December 14-15
- co-located with Agreement Technologies (AT)
- Winter School on AT, December 12-13
- papers published in other conferences are also accepted!
Announcements to attentive agents.

A. Baltag, H. van Ditmarsch, and L.S. Moss.
Epistemic logic and information update.

J.D. Gerbrandy and W. Groeneveld.
Reasoning about information change.

H. van Ditmarsch.
*Knowledge games."
ILLC Dissertation Series DS-2000-06.

W. van der Hoek and M. Pauly.
Modal logic for games and information.

H. van Ditmarsch, J. Halpern, W. van der Hoek, and B. Kooi, editors.