A Logic for Global and Local Announcements

F. Belardinelli

Laboratoire IBISC – Université d’Evry
IRIT Toulouse

joint work with H. van Ditmarsch and W. van der Hoek

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Outline

1 **Background**: logics for (public, semi-private, private) announcements [vDHvdHK15]
   In PAL announcements are
   - **public**: all agents listen to (and are aware of) the announcement
   - **global**: how the new information is processed depends on the model (i.e., public announcements are model transformers)

2 **Goal**: to generalise PAL by weakening **publicity** and **globality**
   - **privacy**: announcements to any subset $A \subseteq Ag$ of agents
   - **locality**: announcements are pointed model transformers

3 **Dynamic Epistemic Logic**: action models allow private announcements, but
   - updated indistinguishability relations are not necessarily equivalences
   - updated models might be strictly larger . . .
   - . . . several problems are undecidable

4 **GLAL**: an extension of PAL supporting both **private** and **local** announcements
   - updated indistinguishability relations are equivalences
   - updated models are normally “smaller” . . .
   - . . . the model checking and satisfaction problems are decidable
The Logic of Global and Local Announcements
Syntax

Let $Ag$ be a set of agents and $AP$ a set of propositional atoms.

**Definition (GLAL)**

Formulas $\phi$ in $L_{glal}$ are defined by the following BNF:

$$\psi ::= p \mid \neg \psi \mid \psi \land \psi \mid CA \psi \mid [\psi]^+_A \psi \mid [\psi]^-_A \psi$$

- $K_a \phi$ is introduced as $C\{a\} \phi$
- $E_A \phi$ is introduced as $\land_{a \in A} K_a \phi$
- $[\psi]^+_A \phi ::= \text{after globally announcing } \psi \text{ to the agents in } A, \phi \text{ is true}$
- $[\psi]^-_A \phi ::= \text{after locally announcing } \psi \text{ to the agents in } A, \phi \text{ is true}$

$$L_{pl} \subseteq L_{el} \subseteq L_{pal^+} \subseteq L_{glal}$$
Formulas in GLAL are interpreted on (multi-modal) Kripke models.

**Definition (Frame)**

A **frame** is a tuple $\mathcal{F} = \langle W, \{ R_a \}_{a \in Ag} \rangle$ where

- $W$ is a set of **possible worlds**
- for every agent $a \in Ag$, $R_a \subseteq 2^{W \times W}$ is an **equivalence relation** on $W$.

A **model** is a pair $\mathcal{M} = \langle \mathcal{F}, V \rangle$ where $V : AP \rightarrow 2^W$ is an assignment to atoms.

- $R_A^C = (\bigcup_{a \in A} R_a)^*$ is the reflexive and transitive closure of $\bigcup_{a \in A} R_a$
- $R(w) = \{ w' \in W \mid R(w, w') \}$ is the $R$-equivalence class of $w \in W$
The **satisfaction set** \( [[\varphi]]_M \subseteq W \) is defined as

\[
[[p]]_M = V(p) \\
[[\neg \psi]]_M = W \setminus [[\psi]]_M \\
[[\psi \land \psi']]_M = [[\psi]]_M \cap [[\psi']]_M \\
[[C_A \psi]]_M = \{w \in W \mid \text{for all } w' \in R^C_A(w), w' \in [[\psi]]_M\} \\
[[\psi^-]_A]_M = \{w \in W \mid \text{if } w \in [[\psi]]_M \text{ then } w \in [[\psi']]_M_{(w, \psi, A)}\} \\
[[\psi^+]_A]_M = \{w \in W \mid \text{if } w \in [[\psi]]_M \text{ then } w \in [[\psi']]_M_{(w, \psi, A)}\}
\]

where **refinements** \( M^-_{(w, \psi, A)} = \langle W^-, \{R^-_a\}_{a \in Ag}, V^-\rangle \) and \( M^+_{(w, \psi, A)} = \langle W^+, \{R^+_a\}_{a \in Ag}, V^+\rangle \) have

- \( W^- = W^+ = W \) and \( V^- = V^+ = V \)
- for every agent \( b \notin A \), \( R^-_b = R^+_b = R_b \); while for \( a \in A \),

\[
R^-_a(v) = \begin{cases} 
R_a(v) \cap [[\psi]]_M & \text{if } v \in R_a(w) \cap [[\psi]]_M \\
R_a(v) \cap [[\neg \psi]]_M & \text{if } v \in R_a(w) \cap [[\neg \psi]]_M \\
R_a(v) & \text{otherwise}
\end{cases}
\]

\[
R^+_a(v) = \begin{cases} 
R_a(v) \cap [[\psi]]_M & \text{if } v \in R^C_A(w) \cap [[\psi]]_M \\
R_a(v) \cap [[\neg \psi]]_M & \text{if } v \in R^C_A(w) \cap [[\neg \psi]]_M \\
R_a(v) & \text{otherwise}
\end{cases}
\]

**Remark**

- for every agent \( a \in Ag \), \( R^-_a \) and \( R^+_a \) are equivalence relations
- \( [\psi]^+_A \) and \( [\psi]^-_A \) are interpreted as local (pointed model) transformers
- the difference between global and local announcements collapse whenever \( A \) is a singleton
Examples: the Muddy Children Puzzle

The model $\mathcal{M}$ for 3 children (red, blue, and green), where no child knows whether she is muddy, can be represented as follows:
Examples: the Muddy Children Puzzle

- Suppose that only red is muddy, i.e., the actual world is \((1, 0, 0)\)
- then, the father **locally** announces to red, green, and blue that at least one child is muddy:
  \(\alpha := m_r \vee m_b \vee m_g\)
- the updated model \(\mathcal{M}^{-}_{(100, \alpha, rgb)}\) is as follows:

  \[
  \begin{array}{cccc}
  (1,1,1) & (1,0,1) & (0,1,1) & (0,1,0) \\
  b & g & r & b \\
  (1,1,0) & (1,0,0) & (0,1,1) & (0,0,1) \\
  g & b & r & g \\
  (0,1,0) & (0,0,1) & (0,1,1) & (0,0,0) \\
  g & b & g & b \\
  \end{array}
  \]

- only the indistinguishability relation for red is updated
- now everybody knows that at least one child is muddy: \((\mathcal{M}, 100) \models [\alpha]_{rgb} E_{rgb} \alpha\)
- the father’s announcement does not make \(\alpha\) common knowledge: \((\mathcal{M}, 100) \not\models [\alpha]^{-}_{rgb} C_{rgb} \alpha\)
- In general, for every world \(s \neq 000\), \((\mathcal{M}, s) \not\models [\alpha]^{-}_{rgb} C_{rgb} \alpha\)
Examples: the Muddy Children Puzzle

- Suppose that the father **globally** announces to red and blue that at least one child is muddy.
- The updated model $\mathcal{M}^+_{(100,\alpha,rb)}$ is as follows:

  \[
  \begin{array}{ccc}
  (0,0,0) & (1,0,0) & (1,1,0) \\
  (0,1,0) & (1,0,1) & (1,1,1) \\
  (0,0,1) & (0,1,1) & (1,1,1) \\
  \end{array}
  \]

  Now the indistinguishability relations for both red and blue are updated and . . .

  . . . they acquire common knowledge that at least one child is muddy: $(\mathcal{M}, 100) \models [\alpha]^{+}_{rb} C_{rb} \alpha$

  - But again the father’s announcement is not enough to make $\alpha$ common knowledge amongst all children: $(\mathcal{M}, 100) \not\models [\alpha]^{-}_{rb} C_{rgb} \alpha$
Examples: Communication Scenario

Consider communication between sender $s$ and receiver $r$ over a reliable channel that is listened to by eavesdropper $e$:

![Graph](image_url)

After $s$ has communicated to $r$ the value of the bit, we obtain the updated model $N(w_1, bit=0, r)$:

Receiver $r$ learns the value of the bit: $N, w_1 / \text{bit}=0 / r K$.

On the other hand, eavesdropper $e$ learns that $r$ knows it: $N, w_1 / \text{bit}=0 / e K w_r / \text{bit}=0 / e K$.
Examples: Communication Scenario

Consider communication between sender $s$ and receiver $r$ over a reliable channel that is listened to by eavesdropper $e$:

$$
\begin{array}{c}
\text{w}_1 \\
\text{0} \\
\text{r, e} \\
\text{1} \\
\text{w}_2
\end{array}
$$

After $s$ has communicated to $r$ the value of the bit, we obtain the updated model $\mathcal{N}_{(w_1, \text{bit}=0, r)}$:

$$
\begin{array}{c}
\text{w}_1 \\
\text{0} \\
\text{e} \\
\text{1} \\
\text{w}_2
\end{array}
$$

Hence, receiver $r$ learns the value of the bit: $(\mathcal{N}, w_1) \models [\text{bit} = 0]_r K_r (\text{bit} = 0)$

On the other hand, eavesdropper $e$ learns that $r$ knows it: $(\mathcal{N}, w_1) \models [\text{bit} = 0]_r K_e K_w r (\text{bit} = 0)$
Examples: Communication Scenario

Compare model $\mathcal{N}$ above with the following bisimilar model $\mathcal{N}'$,
Examples: Communication Scenario

Compare model $\mathcal{N}$ above with the following **bisimilar** model $\mathcal{N}'$,

$$
\begin{align*}
&v'_1 & 0 & r, e & 1 & v'_2 \\
&s, e & 0 & r, e & 1 & s, e \\
w'_1 & 0 & r, e & 1 & w'_2
\end{align*}
$$

However, after communicating to $r$ the value of the bit, the updated model $\mathcal{N}'_{(w'_1, \text{bit}=0, r)}$ is not bisimilar to $\mathcal{N}_{(w_1, \text{bit}=0, r)}$:

$$
\begin{align*}
&v'_1 & 0 & r, e & 1 & v'_2 \\
&s, e & 0 & e & 1 & s, e \\
w'_1 & 0 & e & 1 & w'_2
\end{align*}
$$

In particular, in $w'_1$ eavesdropper $e$ does not learn that $r$ knows the value of the bit: $(\mathcal{N}', w'_1) \not\equiv [\text{bit} = 0]_r Ke Kw_r(\text{bit} = 0)$.

$\Rightarrow$ GLAL is not preserved under standard modal bisimulations.
Comparison with PAL

GLAL is at least as expressive as PAL:

**Proposition**

For all formulas $\phi, \psi$ in PAL, $(M, w) \models [\phi]_w \psi$ iff $(M, w) \models [\phi]_{Ag} \psi$.

By this result we can define a truth-preserving embedding $\tau$ from PAL to GLAL.

**Proposition**

For all formulas $\phi$ in PAL, $(M, w) \models \phi$ iff $(M, w) \models \tau(\phi)$.

Actually, by the example above,

**Theorem**

GLAL is strictly more expressive than PAL, and therefore than epistemic logic.
Comparison with Attentive Announcements

- **Attention-based Announcements** [BDH⁺16]: agents process the new information only if they are paying attention.
- whether they pay attention is handled by a designated set of atoms.
- close relationship with GLAL: in \((N', w'_1)\) although \(r\) processes the new information, agent \(s\) is uncertain about this fact.
- consider adding an ‘attention atom’ \(h_r\) for receiver \(r\) such that \(h_r\) is true in \(w'_1\) and \(w'_2\) but false in \(v'_1\) and \(v'_2\).
- then, the announcement of \(bit = 0\) to \(r\) in \((N', w'_1)\) corresponds to the attention-based announcement wherein sender \(s\) is uncertain as to whether \(r\) is paying attention.

**Differences:**

- [BDH⁺16] models truly private announcements [GG97] (equivalence relations are not preserved), whereas our proposal considers semi-private announcements that do preserve equivalence relations.
- Our announcements are not necessarily public.
Comparison with Semi-Private Announcements

- **Semi-Private Announcements** [GG97, vD00, vdHP06, BvDM08]: after announcing semi-privately $\phi$ to coalition $A$, all agents in $A$ know $\phi$, and the agents in $Ag \setminus A$ know that all agents in $A$ know whether $\phi$.

- In GLAL agents in $Ag \setminus A$ do not necessarily know that all agents in $A$ know whether $\phi$.

- Semi-private announcements can be modeled by refinement $M_{sp}^{(w, \psi, A)}$ according to which $W^{sp} = W$, $V^{sp} = V$, and for $a \in A$,

$$R_{a}^{sp}(v) = \begin{cases} R_a(v) \cap \llbracket \psi \rrbracket_M & \text{if } v \in R_{Ag}^{C} (w) \cap \llbracket \psi \rrbracket_M \\ R_a(v) \cap \llbracket \neg \psi \rrbracket_M & \text{if } v \in R_{Ag}^{C} (w) \cap \llbracket \neg \psi \rrbracket_M \\ R_a(v) & \text{otherwise} \end{cases}$$

- The two frameworks are not directly comparable.
Validities

No complete axiomatisation, but some interesting validities.

- Truthfully announcing a propositional formula $\phi \in \mathcal{L}_{pl}$ entails the knowledge thereof:
  
  $$\models [\phi]_A E_A \phi$$
  $$\models [\phi]_A C_A \phi$$

- Differently w.r.t. PAL, announcements in GLAL cannot be rewritten as simpler formulas. Nonetheless, the following are validities in GLAL:
  
  $$[\phi]_A p \leftrightarrow \phi \rightarrow p$$
  $$[\phi]_A \neg \psi \leftrightarrow \phi \rightarrow \neg [\phi]_A \psi$$
  $$[\phi]_A (\psi \land \psi') \leftrightarrow [\phi]_A \psi \land [\phi]_A \psi'$$

- Further, epistemic operators and nested announcements commute with announcement operators if they refer to the same coalition (but not in general):
  
  $$[\phi]_A^+ C_A \psi \leftrightarrow \phi \rightarrow C_A [\phi]_A^+ \psi$$
  $$[\phi]_A^- E_A \psi \leftrightarrow \phi \rightarrow E_A [\phi]_A^- \psi$$
  $$[\phi]_A^- [\phi']_A^- \psi \leftrightarrow [\phi \land [\phi]_A^- \phi']_A^- \psi$$
  $$[\phi]_A^+ [\phi']_A^+ \psi \leftrightarrow [\phi \land [\phi]_A^+ \phi']_A^+ \psi$$

- Operators $[\phi]_A^+$ and $[\phi]_A^-$ are normal modalities. None of schemes T, S4 and B hold.
A New Notion of Bisimulation

We remarked that GLAL is not preserved under modal bisimulation.

- define $R_A(w, v)$ as: $R_a(w, v)$ iff $a \in A$.

**Definition ($\pm$-Simulation)**

Given models $\mathcal{M}$ and $\mathcal{M}'$, a $\pm$-simulation is a relation $S \subseteq W \times W'$ such that $S(w, w')$ implies

- **Atoms** $w \in V(p)$ iff $w' \in V'(p)$, for every $p \in AP$
- **Forth** for every $A \subseteq Ag$ and $v \in W$, if $R_A(w, v)$ then for some $v' \in W'$, $R'_A(w', v')$ and $S(v, v')$
- **Reach** for every $v, v' \in W$, $a \in Ag$, if $S(v, v')$ then $R_a(w, v)$ iff $R'_a(w', v')$

**Theorem**

If states $s$ and $s'$ are bisimilar, then for every formula $\psi$ in GLAL, $(\mathcal{M}, s) \vdash \psi$ iff $(\mathcal{M}', s') \vdash \psi$. 
Model Checking and Satisfiability

Definition (Model Checking and Satisfiability)

- **Model Checking Problem**: given a finite pointed model \((M, w)\), and formula \(\phi\) in GLAL, determine whether \((M, w) \models \phi\).
- **Satisfiability Problem**: given a formula \(\phi\) in GLAL, determine whether \((M, w) \models \phi\) for some pointed model \((M, w)\).

Theorem

*The model checking problem for GLAL is PTIME-complete.*

Model refinements can be computed in polynomial time.

Theorem

*The satisfiability problem for GLAL is decidable.*

Decision procedure inspired by tableaux for epistemic logic.
Conclusions

Contributions:

- GLAL: a logic for global and local announcements
- strictly more expressive than PAL
- alternative to action models to represent private announcements
- however, not preserved under standard modal bisimulation
- but we have a novel, truth-preserving notion of bisimulation
- the model checking problem is no harder than for epistemic logic
- the satisfiability problem is decidable.

Future Work:

- axiomatisation
- closer comparison with DEL
- more elaborate form of communication (asynchronous, FIFO, LIFO, etc.)
- real-life scenarios and applications
Questions?
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