Unknown Dynamics Decoupling to overcome unmeasurable premise variables in Takagi-Sugeno Observer design

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Abstract— **This paper discusses a new approach to overcome unmeasurable premise variables in observer synthesis for Takagi-Sugeno models. The main idea is based on the decoupling of the nonlinear dynamics in order to manage unmeasured state existing into the membership functions. The obtained structure is a system with membership functions subjected only to measured variables. The stability analysis of the observer is carried out using Lyapunov theory. The observer gains were computed from the resolution of the Linear Matrix Inequality constraints. The present result alleviates the strong conditions assumed in the design of observers for TS systems with unmeasurable premise variables. Simulation results are provided to demonstrate the effectiveness of the proposed approach.**

Keywords: **Unknown Input Observer, Polytopic Systems, Unmeasurable Premise Variables, LMI constraints, Lyapunov Theory.**

I. Introduction

Sensor technologies are more and more forward-thinking. Miniaturization saves space and digitization improves system reliability and robustness. Nevertheless, these technologies remain restricted or insufficient to measure some physical quantities and dynamic states. Hence, researchers have been moving towards the use, since the 1960s, of model-based observers [1], [2] or more recently deep learning techniques [3] as virtual sensors. This new issue allows, in many application fields, to overcome many technical limitations related to sensor deployment, cost and effectiveness.

If the observation problem seems to be solved for a large class of linear systems, the same cannot be claimed for nonlinear systems. In fact, different philosophies have been adopted to overcome the mathematical limitation introduced by time varying parameters and nonlinearities as reviewed in [4], [5]. In particular, input-output linearization of nonlinear systems had opened an active research field towards the generalization of linear control tools to solve the observation problem for nonlinear systems.

Since two decades, Takagi-Sugeno (TS) fuzzy model proposes an attractive way to deal with a wide range of nonlinear system structure for control and estimation purposes. In the polytopic scheme [6], and by using the sector transformation [7], the nonlinear system is transformed in a well-defined compact set to a local linear models smoothly weighted by membership functions. These last depend on the so-called premise variables (PV) which are considered in almost cases to be measurable and then, the problem of designing state observer is straightforward. However, in the opposite case, the problem of designing a state observer should be seriously reconsidered.

Several systematic approaches for observer design for a class of TS systems with unmeasurable premise variables are proposed. In particular, based on Lipschitz hypotheses, a sliding-mode fuzzy observer is presented in [8]. Unfortunately, the low value of the Lipschitz constant, which ensures the existence of observer gains, limits the applicability of this technique. Other approaches are also developed, based on the mean value theorem (MVT) [9] or with the quasi Input to State Stability (qISS) property in [10]. Other ways with nonlinear consequents are also investigated [11]. Here also, even if the Lipschitz condition is relaxed, a new condition is introduced, known as incremental quadratic constraint.

More recently, the immersion techniques have been investigated [4], [12]–[16]. To deals with unmeasurable premise variables, auxiliary dynamics are generated to immerse a given nonlinear system in a new dimension with a suitable structure. In [17], the authors propose a new immersion algorithm for a class of TS systems to transform the weighting functions depending on unmeasured states into a TS system with large dimension where the weighting functions depend only on measured variables. Nevertheless, the immersion technique can lead to a high order problem dimension or simply fail in the immersion transformation due to the recursive derivations. An illustrative example is studied in section II which introduces our starting argumentation for our development in section III.

This work focuses on the observer design for TS systems with state-dependent PV. The main idea is based on the decoupling of the nonlinear dynamics responsible for unmeasurable PV. The observer design problem is reformulated to overcome the restrictive condition of Lipschitz and hence can be applied to a large class of nonlinear systems. In addition, this approach yields to a simple observer synthesis, quite similar to the design of observers for TS systems with measurable PV [18]–[21]. More, the proposed approach estimates both states and unmeasurable premise variables ensuring asymptotic convergence.

The paper is organized as follows. In section II, the problem of designing observers for TS systems with unmeasurable premise variables is highlighted. Sections III and IV state the key result of the paper: a new observer design algorithm and its comparison with the immersion technique is given. Simulations and analysis with existing results are provided in section V. Finally, section VI draws some conclusions and future works.

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II. Problem statement and motivations

Lets consider the following nonlinear system:

$$\begin{cases} \dot{x}(t) &= f(x(t),u(t)) \\ y(t) &= Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control input and $y(t) \in \mathbb{R}^m$ is the output vector; $f(\cdot)$ is a nonlinear function.

Thanks to the sector transformation, the q existing non-linearities can be exactly represented by $r = 2^q$ linear sub-models weighted by membership functions $\mu_i(\nu(t))$, satisfying the convex-sum property in the compact set of the state space, i.e.

$$\sum_{i=1}^{r} \mu_i(\nu(t)) = 1 \quad \text{with} \quad \mu_i(\nu(t)) \geq 0 \tag{2}$$

where $\nu(t)$ is the so-called PV vector depending on system's state. Then, the mathematical formulation of the TS model of system (1) is given by:

$$\begin{cases} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\nu(t)) \left(\bar{A}_i x(t) + \bar{B}_i u(t)\right) \\ y(t) &= Cx(t) \end{cases} \tag{3}$$

A. Classical observer design for TS Systems

If the PV are measured, a classical Luenberger-like observer can be used :

$$\begin{cases} \dot{\hat{x}}(t) &= \sum_{i=1}^{r} \mu_i\left(\nu(t)\right) \left(\bar{A}_i \hat{x}(t) + \bar{B}_i u(t) + \bar{L}_i(y - \hat{y})\right) \\ \hat{y}(t) &= C\hat{x}(t) \end{cases} \tag{4}$$

with $\hat{x}(t)$ is the estimated state vector. The state estimation error, $e(t) = x(t) - \hat{x}(t)$, is governed by:

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\nu(t)) \left(\bar{A}_i - \bar{L}_i C\right) e(t) \tag{5}$$

The stability analysis of the error dynamics (5) may be achieved with sufficient LMI conditions to get the observer gains [23] and [24].

However, if the PV $\hat{\nu}(t)$ are unmeasurable, the previous observer is given by:

$$\begin{cases} \dot{\hat{x}}(t) &= \sum_{i=1}^{r} \mu_i\left(\hat{\nu}(t)\right) \left(\bar{A}_i \hat{x}(t) + \bar{B}_i u(t) + \bar{L}_i(y - \hat{y})\right) \\ \hat{y}(t) &= C\hat{x}(t) \end{cases} \tag{6}$$

In this case, the observer synthesis is not easy and the error dynamics is described by the following pseudo-disturbed TS system:

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i\left(\hat{\nu}(t)\right) \left(\bar{A}_i - \bar{L}_i C\right) e(t) + \delta(t) \tag{7}$$

where the perturbation term is :

$$\delta(t) = \sum_{i=1}^{r} \left(\mu_i(\nu(t)) - \mu_i(\hat{\nu}(t))\right) \left(\bar{A}_i x(t) + \bar{B}_i u(t)\right) \tag{8}$$

and the stability study of the error dynamics becomes more complex with many approaches based on Lipschitz hypotheses [8], MVT [9] or qISS [10] techniques.

B. Immersion techniques: Pros & Cons

As for the principle of the immersion approach, auxiliary dynamics must be added, allowing the transition to a new base as follows:

$$\begin{cases} \dot{v}(t) &= \tilde{A}\left(y(t)\right) v(t) + \tilde{B}\left(y(t)\right) u(t) \\ y(t) &= [C \ 0] v(t) \end{cases} \tag{9}$$

where the corresponding TS form, with measured PV, is:

$$\begin{cases} \dot{v}(t) &= \sum_{i=1}^{r_1} \mu_i\left(\xi(t)\right) \left(\tilde{A}_i v(t) + \tilde{B}_i u(t)\right) \\ y(t) &= [C \ 0] v(t) \end{cases} \tag{10}$$

where r_1 is the number of sub-models and $\xi(t)$ are measured PV (different from $\hat{\nu}(t)$ unmeasured ones).

Hence, the observer governed by equations (5-9) can be synthesized easily.

Let us study the following example:

$$\begin{cases} \dot{x}_1(t) &= -x_1(t) + x_2(t)x_3(t) \\ \dot{x}_2(t) &= x_1(t) - x_2(t) - x_1(t)x_3(t) + 0.5u(t) \\ \dot{x}_3(t) &= -\left(x_1(t) + 1\right) x_3(t) + x_1(t)x_2(t) + u(t) \\ y(t) &= x_2(t) \end{cases} \tag{11}$$

The quasi-LPV form of the previous system (11) is:

$$\begin{cases} \dot{x}(t) &= \begin{bmatrix} -1 & 0 & x_2(t) \\ 1 & -1 & -x_1(t) \\ -x_2(t) & 0 & -1 - x_1(t) \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix} u(t) \\ y(t) &= \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} x(t) \end{cases} \tag{12}$$

The observer synthesis will lead to a TS system with a PV depending on the unmeasured states $x_1(t)$ or $x_3(t)$. Unfortunately, the immersion technique fails to provide a TS system with measurable PV. Definitely, the number of variable changes is unlimited. Hence, the degree of the new variables will increase definitely.

III. Main result

This section aims to decouple the nonlinear dynamics responsible for unmeasurable PV, $\hat{\nu}(t)$. The system (1) and its TS form (3) are reformulated to get a new qLPV structure with a new membership functions $\sigma_i(t)$ with only measurable PV $\xi(t)$. This last, is a subset of the whole PV, $\hat{\nu}(t)$. The PV $\xi(t)$ depends only on outputs $y(t)$ and/or inputs $u(t)$. This leads to a less conservative condition in the observer design of the proposed approach. The resulted system can be written by the following equation:

$$\begin{cases} \dot{x}(t) &= A\big(y(t)\big)x(t) + B\big(y(t)\big)u(t) + Dg\big(x(t),u(t)\big) \\ y(t) &= Cx(t) \end{cases} \tag{13}$$

where D is a constant matrix with appropriate dimension and $g\big(x(t),u(t)\big)$ is a nonlinear function containing sub-dynamics giving rise to an unmeasurable PV.

The corresponding TS structure, with $\xi(t)$ as measured PV, is as following:

$$\begin{cases} \dot{x}(t) &= \sum_{i=1}^{r_2} \sigma_i\big(\xi(t)\big)\big(A_i x(t) + B_i u(t)\big) + Dg\big(x(t)\big) \\ y(t) &= Cx(t) \end{cases} \tag{14}$$

with r_2 is the number of sub-models of the new structure such that $r_2 < r$.

IV. Unknown Dynamics Observer design

The present section introduces the design of the unknown input observer considering the qLPV model (14) and the observer structure proposed in [18] and [19]. The conditions for the observer's asymptotic convergence are derived from Lyapunov analysis and the corresponding LMI optimisation problem is stated.

In this context, the following non restrictive assumptions are considered:

Assumption 1: In what follows, it is supposed that:

— the state vector $x(t)$ is supposed bounded,

— $rank(CD) = rank(D)$ [22],

— the pair $\big((A(\xi(t)), C\big)$ is observable, al least detectable, for all $\xi(t)$ in the compact set.

Remark 1: If the third condition in assumption (1) is not satisfied, it is possible to add sub-dynamics in matrix $A(y(t))$ and to subtract them in function $g(x(t),u(t))$ in a way that, at the end, the new structure remains observable without affecting the original system (1). In such case, for the design of the observer, the last modified matrices $A(y(t))$ and D must be considered.

A. Observer design

Let us consider the following nonlinear observer for the new structure (14), [18] :

$$\begin{cases} \dot{z}(t) &= N\big(\xi(t)\big)z(t) + G\big(\xi(t)\big)u(t) + L\big(\xi(t)\big)y(t) \\ \hat{x}(t) &= z(t) - Hy(t) \end{cases} \tag{15}$$

where the observer's matrices $N_\xi \in \mathbb{R}^{n\times n}$, $G_\xi \in \mathbb{R}^{n\times p}$, $L_\xi \in \mathbb{R}^{n\times m}$ and $H \in \mathbb{R}^{n\times m}$ are to be determined to ensure asymptotic convergence of the estimation error. Here, and for the rest of the paper, we use the abbreviated notation $M\big(\xi(t)\big) = M_\xi$.

Also, N_ξ and L_ξ have the same quasi-LPV form as the matrix A_ξ and may be written in a TS form as follows:

$$N_\xi = \sum_{i=1}^{r_2}\sigma_{i_\xi}N_i,\ G_\xi = \sum_{i=1}^{r_2}\sigma_{i_\xi}G_i,\ L_\xi = \sum_{i=1}^{r_2}\sigma_{i_\xi}L_i \tag{16}$$

We recall here that the weighing functions σ_{i_ξ} depend only on the outputs.

According to equations (13), (14) and (15), the state estimation error is given by:

$$e(t) = x(t) - \hat{x}(t) = \underbrace{(I + HC)}_{P}x(t) - z(t) \tag{17}$$

Hence, the state estimation error is governed by the following differential equation:

$$\begin{aligned} \dot{e}(t) &= PA_\xi x(t) + (PB_\xi - G_\xi)u(t) + PDg\big(x(t),u(t)\big) \\ &\quad -N_\xi z(t) - L_\xi Cx(t) \\ &= N_\xi e(t) + \big(PA_\xi - N_\xi P - L_\xi C\big)x(t) \\ &\quad +\big(PB_\xi - G_\xi\big)u(t) + PDg\big(x(t),u(t)\big) \end{aligned} \tag{18}$$

B. Convergence study

Under the following conditions:

$$PB_\xi - G_\xi = 0 \tag{19}$$

$$PA_\xi - N_\xi P - L_\xi C = 0 \tag{20}$$

$$PD = 0 \tag{21}$$

the estimation error dynamics will be reduced to:

$$\dot{e}(t) = N_\xi e(t) \tag{22}$$

To obtain the observer gains, the linear matrix equalities (19), (20) and (21) must be satisfied. In that cas, the asymptotic convergence of the estimation error's vector $e(t)$ is governed by the behavior of the matrix N_ξ in (22).

First, the matrix H is computed from (21):

$$\begin{aligned} (I + HC)D = 0 \quad &\Leftrightarrow \quad HCD = -D \\ &\Leftrightarrow \quad H = -D(CD)^+ \end{aligned} \tag{23}$$

where: $(CD)^+ = \left[(CD)^T(CD)\right]^{-1}(CD)^T$ is the left pseudo-inverse of the matrix CD. This equality constraint is equivalent to the rank condition in assumption (1).

After computing the matrix H, the matrix P is computed and replaced in the equality (20) which, after a simple calculation leads to:

$$\underbrace{PA_\xi}_{\Gamma_\xi} - N_\xi - \underbrace{(N_\xi H + L_\xi)}_{K_\xi}C = 0 \quad \Leftrightarrow \quad N_\xi = \Gamma_\xi - K_\xi C$$

Then, the state estimation error's dynamics becomes:

$$\dot{e}(t) = \big(\Gamma_\xi - K_\xi C\big)e(t) \tag{24}$$

Let us consider the following quadratic Lyapunov function to address the stability analysis:

$$V\big(e(t)\big) = e^T(t)Xe(t), \quad X = X^T > 0 \tag{25}$$

whose time derivatives $\dot{V}(e(t))$ leads to:

$$\begin{array}{rcl}\dot{V}\left(e(t)\right) & = & e^T(t)\left(N_\xi^T X + X N_\xi\right) e(t) \\ & = & e^T(t)\left((\Gamma_\xi - K_\xi C)^T X + X(\Gamma_\xi - K_\xi C)\right) e(t) \\ & = & e^T(t) \sum_{i=1}^{r_2} \sigma_{i_\xi} \left((\Gamma_i - K_i C)^T X \right. \\ & & \left. + X\,(PA_i - K_i C)\right) e(t) \end{array} \quad (26)$$

By considering the following change of variable $\bar{K}_{i_\xi} = XK_{i_\xi}$ for $i = 1, \cdots, r_2$, we have:

$$\dot{V}\left(e(t)\right) < 0 \Leftrightarrow \Gamma_{i_\xi}^T X^T - C^T \bar{K}_{i_\xi}^T + X^T \Gamma_{i_\xi} - \bar{K}_{i_\xi} C < 0, \ \ \forall i \quad (27)$$

If these conditions hold then $\dot{V}\left(e(t)\right) < 0$.
Hence, from (27), an optimization problem is derived with a compact set of the following LMIs constraints to be solved:

$$A_i^T P^T X - C^T \bar{K}_{i_\xi}^T + XPA_i - \bar{K}_{i_\xi} C < 0, \text{ for } i = 1, \dots, r_2 \quad (28)$$

Finally, the observer design procedure is summarized as following:

1) Compute matrix H from (21) and matrix P from (17),
2) Deduce the matrix G_ξ from (19),
3) Solve the LMIs in (28) and compute the observer gains K_{i_ξ},
4) Deduce N_i and L_i from (29),
5) Use (16) to determine the observer's matrices.

$$\begin{array}{rcl} K_{i_\xi} & = & X^{-1}\bar{K}_{i_\xi} \\ N_i & = & \Gamma_{i_\xi} - K_{i_\xi} C \\ L_i & = & K_{i_\xi} - N_i H \end{array} \quad (29)$$

The unknown input observer allows to estimate the whole of state vector $x(t)$ but does not give information about unknown dynamics, that is why we need to proceed into a reconstruction of the unknown dynamics based on estimated states and output derivatives. To estimate the state and output time derivatives, a High-Order sliding mode differentiator is used. For more details on this type of signal differentiation algorithm, please refer to [26].

The nonlinear dynamics, which we decouple, can be reconstructed by the following approach. From the state space of the system, one can write:

$$\dot{\hat{x}}(t) = A\left(y(t)\right)\hat{x}(t) + B\left(y(t)\right)u(t) + Dg\left(\hat{x}(t), u(t)\right) \quad (30)$$

Multiplying this last equation by matrix C and after slight calculations, one can get:

$$g\left(\hat{x}(t), u(t)\right) = (CD)^+\left(\dot{y}(t) - CA(y)\hat{x}(t) - CB(y)u(t)\right) \quad (31)$$

V. Simulations and Discussions

In this section we will study two different examples. The first example is the oscillator studied in [17] where the immersion technique and the proposed approach converge. The second example is a system which highlights the advantages of the proposed approach while the immersion approach fails.

The gains of the Unknown Dynamics Observer (UDO) are computed from the equations (23)-(29) applied to both examples.

A. Illustrative Example 1

Consider the Lorenz system given by the following state space:

$$\begin{cases} \dot{x}_1(t) & = & -10\left(x_1(t) - x_2(t)\right) \\ \dot{x}_2(t) & = & 28x_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) & = & -\frac{8}{3}x_3(t) + x_1(t)x_2(t) \\ y(t) & = & x_2(t) \end{cases} \quad (32)$$

The synthesis of the proposed observer implies the new structure (13) of the system with:

$$A\left(y(t)\right) = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -1 \\ y(t) & 0 & -\frac{8}{3} \end{bmatrix}, B\left(y(t)\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$D = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

where $g\left(x(t), u(t)\right) = -\left(x_1(t) - 1\right)x_3(t)$ and $C = [0 \ \ 1 \ \ 0]$. With respect to remark 1, an additional dynamics $\pm x_3(t)$ has been added to the system state space $\left(\dot{x}_2(t)\right)$ to ensure observability of the targeted system.

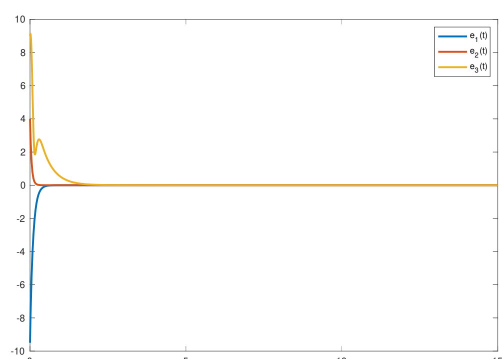

Fig. 1: State estimation errors, UDO technique.

By using *Sedumi* & *Yalmip* [25] LMI toolbox, one can solve (28) and (29) and find the following observer gains:

$$N_1 = \begin{bmatrix} -10 & 0.7 & 0 \\ 0 & -21.37 & 0 \\ 50 & 146.07 & -2.67 \end{bmatrix},$$
$$N_2 = \begin{bmatrix} -10 & 6.16 & 0 \\ 0 & -17.01 & 0 \\ 50 & -137.55 & -2.67 \end{bmatrix}$$

and

$$L_i = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}, H = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \text{ and } G = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Taking $x_0 = [0.5 \quad 4 \quad -1]^T$, simulation results are shown in Fig. 1.

Fig. 1 shows results of the proposed unknown dynamics TS observer, synthesized under the same conditions for the observer based on immersion approach. The observer convergence is very good and no need to Lipschitz assumption neither to the immersion technique to obtain correct convergence of the state estimation.

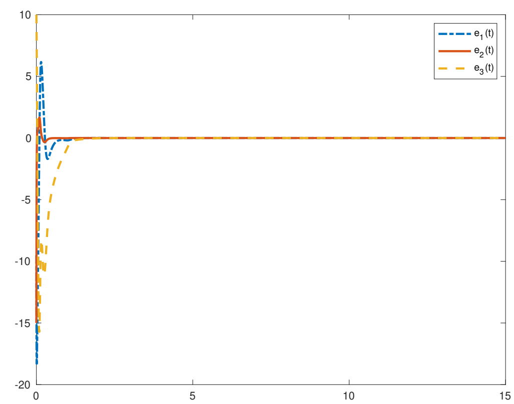

Fig. 2: State estimation errors, Immersion technique.

In Fig. 2 are depicted the estimation errors given by the observer based on the immersion technique, on the same system. The calculated gains here are exactly the ones given in paper [17].

B. Illustrative Example 2

Consider the example (11) where the immersion technique fails.

The system states are depicted in Fig. 3 with external excitation of sinusoidal form.

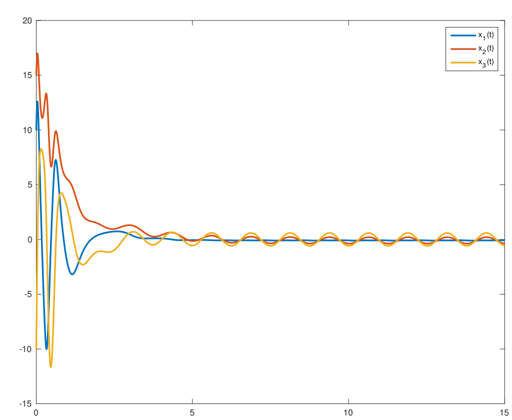

Fig. 3: System states evolution.

The synthesis of the proposed observer implies the new structure (13) of the second example with:

$$A\left(y(t)\right) = \begin{bmatrix} -1 & 0 & y(t) \\ 1 & -1 & 0 \\ -y(t) & 0 & -1 \end{bmatrix}, B\left(y(t)\right) = \begin{bmatrix} 0 \\ 0.5 \\ 1 \end{bmatrix}$$

and

$$D = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

where $g\left(x(t), u(t)\right) = -x_1(t)x_3(t)$ and $C = [0 \ 1 \ 0]$. The input signal is $u(t) = 2\sin(2t) * \sin(20t)$. The observability matrix O of example 2 is given by:

$$O = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ -2 & 1 & y(t) \end{bmatrix}$$

otherwise, the system is uniformly observable if and only if $y(t)$ is non zero. One can easily avoid the related singularities by adding the same dynamics, for example $\pm x_3(t)$, to the state space dynamics $\dot{x}_2(t)$ and $\dot{x}_3(t)$. In this case the rank of the observability matrix O is output free.

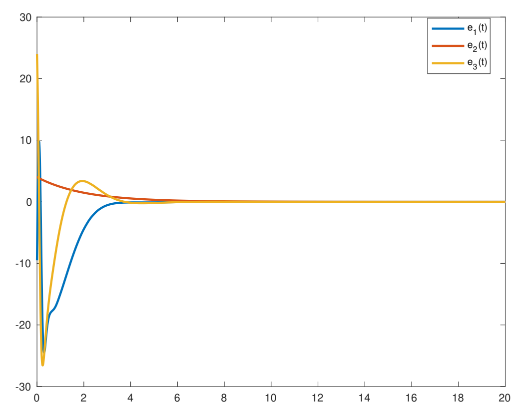

Fig. 4: State estimation errors, UDO technique.

By using *Sedumi* & *Yalmip* LMI toolbox, one can solve (28) and (29) and find the following observer gains:

$$N_1 = \begin{bmatrix} -1 & -0.00005 & 50 \\ 0 & -0.5 & 0 \\ -51 & 0.00009 & -1 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -1 & -0.00005 & -50 \\ 0 & -0.5 & 0 \\ 49 & -0.00008 & -1 \end{bmatrix}$$

and

$$L_i = \begin{bmatrix} (-1)^i 50 \\ 0 \\ 0 \end{bmatrix}, H = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix} \text{ and } G = \begin{bmatrix} 0 \\ 0 \\ 0.5 \end{bmatrix}$$

Taking $x_0 = [0.5 \quad 4 \quad -1]^T$, simulation results are shown in Fig. 4.

Fig. 5 presents the evolution of the unknown dynamics estimation error. By subtracting equations (13) and (31), this last error is governed by the following equation:

$$e_g(t) = g\left(x(t)\right) - g\left(\hat{x}(t)\right) = -(CD)^{+}CA(y)e(t) \qquad (33)$$

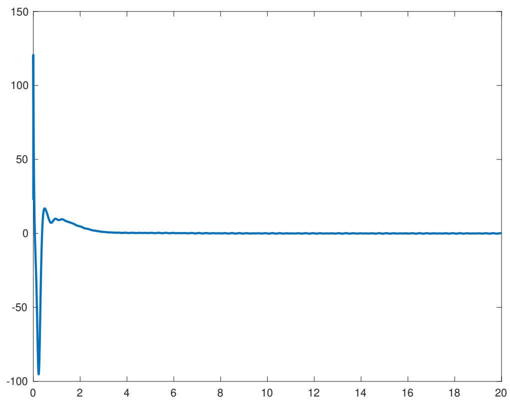

Fig. 5: Unknown dynamics estimation errors.

It is clear, that if the error of the state estimation converge asymptotically to zero, the error estimation of the unknown dynamics also converge to zero.

VI. Conclusions

In the present work, we addressed a new approach to overcome the unmeasurable premise variables in observer synthesis for Takagi-Sugeno models. The approach consists in transforming a nonlinear system into a new structure by isolating a part of system dynamics. This part is directly responsible of the occurrence of unmeasurable premise variables in the observer design. After that, and under some conditions, an unknown dynamics observer is used to estimate the state vector with membership function depending only on the output vector under less conservative synthesis.

Numerical results confirm the overview offered by the proposed approach compared to the classical or immersion approaches. Future results will concern the generalization of the proposed approach when the matching condition in not met.

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