MUTIPLE-GRADIENT DESCENT ALGORITHM FOR PARAMETRIC IDENTIFICATION OF A POWERED TWO-WHEELED VEHICLES

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Abstract— **Powered Two-Wheeled vehicles (PTWv) are an increasingly popular means of transport. The cost and the risks of the development phase of this vehicles has to be diminished in order to ensure acceptable levels of comfort and safety for riders upstream of hazardous driving situations. It is required to study motorcycle while riding in cornering and lane-keeping to interpret the dynamics behavior. Thus, we need to obtain dynamic model that tightly adjusts to the real lateral behavior of the motorcycle, in the way that it will lead to precise simulation and experimental results.**

A technique for cascade identification of parameters based on data and optimization algorithm, is presented here. This methodology makes profit of the possibility given by this type of algorithm for solving multiple objectives function consecutively. After the identification method is outlined, simulations and experimental results are presented in order to confirm the accuracy of the parameters estimation under the persistent condition of the inputs.

1. INTRODUCTION

There are a broad range of mathematical models that are often used in engineering applications, trustworthy models are therefore crucial to describe the actual process before synthesizing a control law or an observer.

However, even if a parametric model can be derived, the parameters value are not so. It is therefore necessary to identify the value of these parameters. We must first find the right compromise between the fidelity and simplicity of the model to be used in the parametric identification process.

Optimization methods are of major importance in parameter identification problems. One can distinguish two main classes of techniques to solve an optimization problem, gradient methods require the calculation of the gradient of the function to be minimized and stochastic methods [**?**] which have a major place in non-differentiable optimization. The choice of a method depends on the nature of the problem (differentiable, non-differentiable, ...). The gradient methods are simple to implement and often yield good results. For this reason, they are generally used in practical applications ([**?**], [**?**]).

This article is focused on the identification of a mathematical model of a two-wheeled vehicle from measurement carried out on a motorcycle benchmark software. Thus, we have first developed an estimation strategy based on the gradient method applied on a two bodies Sharp model [**?**]. It solves an algorithm for multi-objective optimization problems based on initial values close to the global minima. This model contains several parameters, some of which are unknown and hence have to be identified. The complexity of the model and representation suggests an iterative cascade algorithm to solve step-by-step the objective-functions, so we manage the equations in order to avoid algebraic loop. Multi-objective techniques applied to model identification have achieved great results in many cases, as shown in ([**?**], [**?**] and [**?**]). First, a general study of the Sharp model is conducted under the BikeSim software. This study allowed us to draw up a balance sheet of the various knowledge and elements to be included in the identification problem, some geometric equations are also added in order to simplify the complexity of the resolution problem, while ensuring a certain precision and realism.

To the knowledge of the author, works on the parametric identification of the PTWv have been very few, main research were achieved without considering the physical model of the two-wheeled vehicle. In [**?**], the author considers an autoregressive model of motorcycle to estimate the state space model of lateral dynamics without identifying parameters. This method allowed statistical estimation of state space models under manual control. There are other research axes which are concerned with the identification of the controller parameters to stabilize the two-wheeled vehicle, in ([**?**],[**?**]) the authors consider the rider control as a linear PID controller, this control model is fitted to data and aim to mimic realistic rider control behavior.

The main difficulty in identifying the values of parameters is the complexity of the model and the choice of the persistent input to well exciting the dynamic of all the parameters. In the present contribution we will provide a procedure to identify parameters of a linear gray-box motorcycle model.

However, our approach is, to the best of the authors knowledge, new in the sense that we can both identify values of parameters and also state space vector of the motorcycle in order to estimate the cornering motion and the unstable mode of the vehicle without using observer. The paper is organized as follows. Section II, describes the Out-of-plane motion of PTWvs. Section III, presents the identifiability and motorcycle model identification with an optimization method to identify parameters of the Sharp model. Section IV, provides some simulation results, present the vibration modes and the lateral stability of the identified model. Section V, analyze the estimated model. Finally, section VI concludes the paper.

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2. TWO-WHEELED LATERAL DYNAMICS DESCRIPTION

2..1. Motorcycle Description

The model developed by Sharp in 1971 [?] is a founding study that has contributed to the modeling of motorcycles ([?], [?]). This model allows to simulate the 4 DoF which are the lateral displacement, the roll and the yaw motion, as well as the steering dynamics. The author represented the motorcycle as a set of two bodies linked by the steering mechanism (figure ??).

1. The first G_f body is the front frame, consisting of the front wheel, the fork, the handlebars and the couplings.

2. The second body G_r is the rear frame, consisting of the main structure, engine-gearbox assembly, fuel tank, seat, rear swingarm, rear wheel and a rigid jumper.

3. The rider is rigidly connected to the rear body, and its motion is not taken into account.

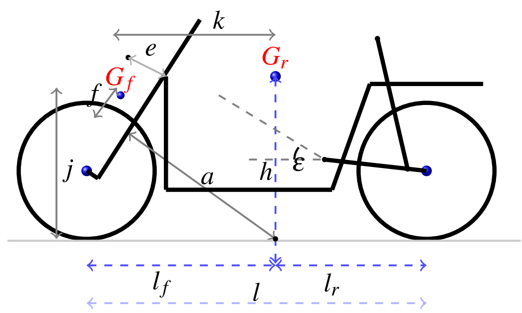

Fig. 1. The motorcycle geometrical description

2..2. Out-of-Plane motion

Here, the lateral dynamics of a motorcycle are due essentially to the effect of lateral forces from the front and rear wheels (F_{yf} and F_{yr}) and the yaw and roll motions under rider's steering actions. These motions are expressed by the following equations :

- Lateral motion

$$M\dot{v}_y + \theta_1\ddot{\psi} + \left(M_f j + M_r h\right)\ddot{\phi} + \theta_2\ddot{\delta} + M v_x\dot{\psi} = F_{yf} + F_{yr} \quad (1)$$

- Yaw motion

$$\theta_1\left(\dot{v}_y + v_x\dot{\psi}\right) + \theta_4\ddot{\phi} + \theta_3\ddot{\psi} + \theta_5\ddot{\delta} - \theta_6 v_x\dot{\phi} - \theta_7 v_x\dot{\delta} = l_f F_{yf} - l_r F_{yr} \quad (2)$$

- Roll motion

$$\left(M_f j + M_r h\right)\dot{v}_y + \theta_8\ddot{\phi} + \theta_4\ddot{\psi} + \theta_9\ddot{\delta} - \theta_{10} v_x\dot{\psi} - \theta_{11} v_x\dot{\delta} = \left(M_f j + M_r h\right) g\sin(\phi) + \theta_{12}\sin(\delta) \quad (3)$$

- Steering motion

$$\theta_2\dot{v}_y + \theta_9\ddot{\phi} + \theta_5\ddot{\psi} + \theta_{13}\ddot{\delta} + \theta_{11} v_x\dot{\phi} - \theta_{14} v_x\dot{\psi} - \theta_{15}\dot{\delta} = \theta_{12}\sin(\phi) + \theta_{12}\sin(\varepsilon)\sin(\delta) - \eta F_{yf} + \tau \quad (4)$$

These equations are comprehensive and not all states and parameters are known, geometric parameters (l_r, l_f, l, h, η, ε), and inertial parameters($M = M_f + M_r$) are usually known.

θ_i : are parameters to be identified. Some of them are depicted in figure (??).

In most cases, sensors allow to measure different state variables. For our PTWv, we measure the following state:

- the roll and yaw rates $\dot{\phi}$, $\dot{\psi}$,
- the longitudinal velocity v_x,
- the steering angle δ and its time-derivative rate $\dot{\delta}$ and $\ddot{\delta}$,
- the lateral acceleration which verifies the equation : $Ma_y = F_{yf} + F_{yr}$.

Let us first investigate what we know about the motorcycle.

$$\begin{cases} k = (a+e)\cos(\varepsilon) - f\sin(\varepsilon) \\ j = (a+e)\sin(\varepsilon) + f\cos(\varepsilon) \\ a = \frac{l_f}{\cos(\varepsilon)} - \eta \\ l = l_f + l_r \\ Z_f = -\frac{M}{l} l_r g \\ Z_r = -(M_f + M_r)g + Z_f \end{cases} \quad (5)$$

3. MOTORCYCLE MODEL IDENTIFICATION

The parametric identification consists in determining the best values of parameters in sense of an appropriate criterion, this is an optimization problem whose resolution quickly becomes arduous as soon as the number of parameters to be identified increases. Moreover, the parameter identification is a challenge in terms of choosing the estimation method and the shape of the model.

The purpose of this section is to present a method for identifying parameters of the 2-body model using all the information that can be harvested on the model as well as some test scenarios on the motorcycle.

3..1. Identifiability

Theoretical models derived from physical laws describe well the behavior of the systems to be identified, but these models usually suffer from problems of identifiability [?].

The objective of identifiability is to verify that two vectors of different parameters do not lead to the same input-output behavior, it requires that different parameter values give different model outputs, which results in:

$\exists$ (u, x_0) $\in$ (U $\times$ $\mathbb{R}^{n_x}$) such that

$$y_M(t,u,\theta) = y_M(t,u,\theta^*), \ \forall \in t, \ \ \theta = \theta^*$$

where θ and θ^* belong to $\mathscr{P}$, a set of admissible parameters. This intrinsic property of the model is a necessary condition to ensure that the adjustment procedure leads to a single value of the parameter θ and thus to reliable model predictions.

For more detail on identifiability please refer to ([?], [?],[?] and [?]).

According to Ollivier, F ([?] [?]), it is of interest to test numerically the local identifiability.

Theorem 1: [?] A linear system with zero initial conditions (A, B and C : the states-matrix) is identifiable if the application

$$\rho : \theta \mapsto [C(\theta)B(\theta), C(\theta)A(\theta)B(\theta), \cdots, C(\theta)A(\theta)^{n-1}B(\theta)]$$

is invertible on a dense open set.

It is locally identifiable if and only if the application ρ is locally invertible in the neighborhood of any point of a dense open set.

One can test the existence of a left inverse for ρ by a standard basic computation, but this is only a sufficient condition of identifiability.

If ρ is invertible to the left, its determinant can not be zero, it is easy to verify: if ξ is the inverse to the left of ρ, we have

$$det(\rho)det(\xi) = 1 \neq 0$$

3..2. Multi-Objective Optimization

In engineering problems, it is a common issue to deal with situations that require multi-criteria optimization. Due to this fact, addressing these problems from the standpoint of classical optimization could be insufficient.

Any multi-objective optimization problem can be stated as:

$$\min_{x \in R^n} \mathscr{C}(\theta) = [C_1(\theta), C_2(\theta) \dots C_9(\theta)] \tag{6}$$

Generally, it will not be possible to find a solution θ that satisfies all requirements and minimizes all the cost function $\mathscr{C}(\theta)$ in the same time. Scenarios of test are chosen to well excite the parameter to be identified. So, this method allows an identification in cascade even if we have more parameters then equations.

it is necessary to make some approximations to start the identification algorithm and to decouple equations with respect to parameters. Under some hypothesis, we know that $M_r \gg M_f$ and $h \simeq j$, thus:

$$\begin{cases} M = M_f + M_r \\ Mh = M_f j + M_r h \end{cases} \tag{7}$$

We manipulate the system of equations (**??-??-??-??**) to extract each unknown parameter separately in order to identify the parameters in cascade i.e the first system allows to identify θ_1 then the second parameter θ_2 is identified based on the knowledge of θ_1 and so on.

thus:

$$\begin{cases} y_1 = f(y, \theta_0, \theta_1) \\ y_2 = f(y, \theta_0, \theta_1, \theta_2) \\ y_3 = f(y, \theta_0, \theta_1, \theta_2, \theta_3) \\ \dots \\ \dots. \\ y_i = f(y, \theta_0, \theta_1, \theta_2, \dots, \theta_j) \end{cases} \tag{8}$$

Where:

- y_i : measured outputs of the motorcycle.
- θ_0 : initial value of parameters.
  With : $i = 1, \dots, 9$ $j = 1, \dots, 15$

$$\theta = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_j \end{pmatrix} \tag{9}$$

Please refer to Appendix for the expressions of y_i and θ_j.

To solve the stated above, any Multi-Objective optimizer can be used. In this work, the gradient algorithm has been chosen.

3..3. Identification of vector parameters θ by the multiple-gradient method

The aim of this part is the identification of the parameters of a motorcycle body model while setting up an algorithm for the automatic estimation of parameters. The processing of the input / output data is done through a recursive method based on the optimization by the gradient algorithm.

The advantages of gradient method is that every iteration is inexpensive and does not require second derivatives. **Approach**: The methodology used for the identification of the parameters is as follows :

- First, we use the parameters initialized from the appropriate data close to our motorcycle in terms of characteristics, so we have determined the priori value of parameters.
- Secondly, we seek the persistent input which excites all the dynamics of the motorbike to have the measured data. This is possible in simulation, but in practice this can be difficult to reproduce.
- Finally, the gradient method is used to estimate the vector of parameter in cascade. We consider the lateral motion of the powered two-wheeled vehicle, as a two bodies, the riders torque is the input of the system.

Recording outputs on the system will allow us to identify the model parameters described in the equations (**??**).

The gradient method is an iterative optimization algorithm for solving problems of the form (**??**) with the search directions defined by the gradient of the function at the current point. The criterion chosen is a quadratic function:

$$\begin{cases} \text{criteria} : C_i(t_k) = \frac{1}{2} . \sum (y_{i_m}(t_k) - y_i(t_k))^2 \\ \\ \text{sensibility} : S_i(t_k) = \frac{dy_i}{d\theta_j} \\ \\ \text{gradient} : g_i(t_k) = \sum (y_{i_m}(t_k) - y_i(t_k)) . S_i \end{cases} \tag{10}$$

Where:

- y_{i_m}: Measured outputs.
- y_i : Model output approximation at the instant t_k.
- t_k : Moments of measurement.
  with : $i = 1, \dots, 9$ $j = 1, \dots, 15$

$$\theta = \begin{pmatrix} \theta_1 \\ \vdots \\ \theta_{15} \end{pmatrix} \tag{11}$$

In our case, the vector $\theta_y = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6, \theta_7, \theta_8, \theta_{15})^T$ can be identified from different kind of riding scenario, after the identification algorithm, we deduce the other motorbike parameters :

$$\theta_x = (k, e, a, f, j, I_{fz}, i_{fy}, i_{ry}, \theta_9, \theta_{10}, \theta_{11}, \theta_{12}, \theta_{13}, \theta_{14})^T$$

which are combination of the identified one. The following system is used to deduce θ_x :

$$\begin{cases} k = \frac{\theta_1}{M_f} \\ e = \frac{\theta_2}{M_f} \\ a = \frac{l_f}{\cos(\varepsilon)} - \eta \\ f = \frac{a+e}{\tan(\varepsilon)} - \frac{k}{\sin(\varepsilon)} \\ j = (a+e)\sin(\varepsilon) + f\cos(\varepsilon) \\ I_{fz} = \frac{\theta_5 - M_f ek}{\cos(\varepsilon)} \\ i_{fy} = \frac{\theta_7 . R_f}{\sin(\varepsilon)} \\ i_{ry} = \frac{\theta_6 - i_{fy}}{R_f} R_r \\ \theta_{11} = -\frac{-\theta_7}{\tan(\varepsilon)} \\ \theta_{10} = -M_f j + M_r h + \theta_6 \\ \theta_{14} = -M_f . e + i_f y / R_f . \sin(\varepsilon) = -\theta_2 - \theta_7 \\ \theta_9 = M_f . e . j + I_f z . \sin(\varepsilon) \\ \theta_{13} = I_{fz} + M_f . e^2 \\ \theta_{12} = M_f eg - \eta Z_f \end{cases} \tag{12}$$

This method depends heavily on the input excitation, for example, θ_1 can be identified only from lateral dynamics when the roll and yaw will be well excited.

As an example, we identify the value of θ_1 via the gradient method from equation of lateral motion (**??**) we know that $Ma_y = F_{yf} + F_{yr}$, we choose a scenario when the second derivative of steering angle was null $\ddot{\delta} = 0$ i.e (The curve of δ is a ramp) thus, from equation 1 of the system (**??**) we find :

$$M\dot{v}_y + \theta_1 \ddot{\psi} + (Mh)\ddot{\phi} + Mv_x \dot{\psi} = Ma_y$$

We consider in this case : $y_1 = \ddot{\psi}$

$$\begin{cases} y_1 = -\frac{(Mh\ddot{\phi} + Mv_x\dot{\psi} + M\dot{v}_y - Ma_y)}{\theta_1} \\ \\ S_1 = \frac{d\ddot{\psi}}{d\theta_1} = \frac{(Mh\ddot{\phi} + Mv_x\dot{\psi} + M\dot{v}_y - Ma_y)}{\theta_1^2} \\ \\ g_1 = \sum (y_1 mesure(t) - y_1(t)).S_1 \end{cases} \tag{13}$$

We applied the algorithm to find the value of the first parameter θ_1, then to deduce the geometrical parameter $k = \frac{\theta_1}{M_f}$. The algorithm runs as follows:

1) From an initial starting point $\theta_1(t_0)$, we calculate the criterion $C_1(t_0)$ and the gradient $g_1(t_0)$, this gradient indicates the direction of the largest increase of y_1 .
2) Calculating a new value of
$\theta_1(t_k) = \theta_1(t_{k-1}) - \alpha g_1(t_{k-1})$
Thus, we calculate a new value of $C_1(t_k)$ and $g_1(t_k)$ taking $\theta_1(t_k)$ (the point on the direction of the previous gradient away by step α)
3) If the second criterion is smaller, keep the new parameter value of the corresponding $\theta_1(t_k)$. Increase α for efficiency. And increment the counter k.
4) Otherwise keep the old value of $\theta_1(t_{k-1})$ and reduce α to seek a nearest local minimum.
5) Evaluate various stopping criteria for exit loop: accuracy on the criteria, the gradient, maximum number of effective iteration, tolerance between the last two values of θ.

4. Result and Simulation

4..1. Parameter Estimation and Model Validation

In order to obtain data that can be employed in adjusting the physical parameters, the designed experiments simulate such short maneuvers. Thus, each system of equation has been excited separately and, after that excitation, the motorcycle reacts and the appropriate parameter is well excited. After identification, these parameters are used to calculate the response of the identified model and test stability. We excite the actual system and the identified model with the same steering torque input (figure **??**) in order to validate the identification method, so figures show the lateral models found by the optimization algorithm and the error between real system and estimated response of the model.

- For pneumatic parameters that are influenced by tire types and road condition, the typical values presented in ([**?**]) are used while the other accessible parameters are taken from data sheet of the motorcycle.
- By the nature of the system, the model is unstable. It was necessary to add a corrector to guarantee the stability of the system so a state feedback deduced by the placement of poles is used.
- The choice of the excitation input by simulation so that the output will be well excited to allow identification.
- Note that the prior inertial parameters are computed from *BikeSim* using the Huygens theorem.

To validate the model, it is sufficient to retrieve the input data measurements acquired during the experiment for exciting the identified model to observe the outputs responses, thus comparing it with the measured output. I chose a scenario where the motorcycle was turning on the mini-roundabout to better check the variation of the roll angle.

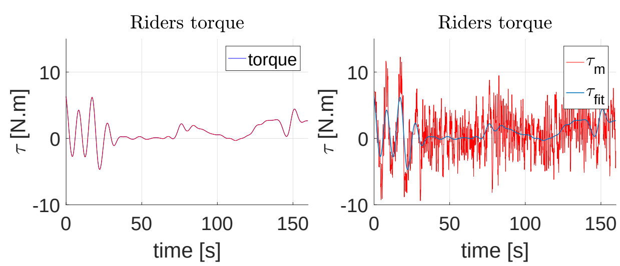

Fig. 2. Riders torque.

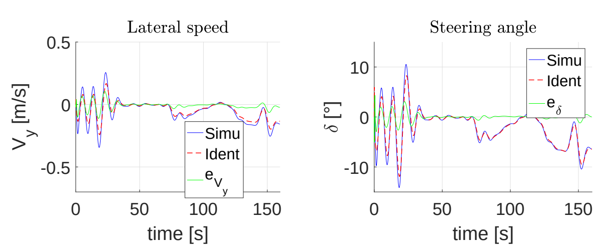

Fig. 3. Lateral speed-Steering angle.

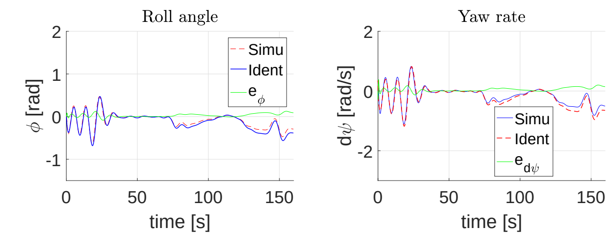

Fig. 4. Roll angle-Yaw rate.

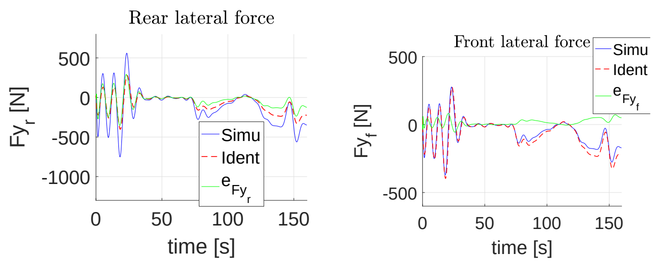

Fig. 5. Rear lateral force-Front lateral force.

From graphs (**??**-**??**-**??**-**??**) of the state estimates and their estimation errors, we verify many results that are consistent with response of actual system with a very close look. Figures (**??**) show the evolution of the riders torque τ taken as a persistent excitation signal, applied in the proposed estimation algorithm to identify parameters and states variables of the motorbike, in figures (**??**) we see that the lateral speed and the steering angle are well estimated from the resulting identified parameters with an error around zero, figures(**??**) plot the roll and yaw rate for the motorbike and their corresponding state estimation, then in figures(**??**), the rear and front lateral forces are given as a comparison between the actual and identified state. We obtain results of simulation that converges well to the output of the state space model of motorcycle.
The method works. However the accuracy of the identified values depends on the initial parameter value and the coefficient α in each iteration of the algorithm.
As can be seen, The cornering behavior of the motorbike is well identified, these indicate that the proposed parameter and state estimation algorithm is effective.

4..2. Vibration modes and lateral stability of the identified model PTWv

Motorcycles, are a complex mechanism that gather a set of actions and physical phenomena during motorcycle driving [**?**]. It is very important to know how a motorcycle reacts, to be able to transmit the right orders to keep the control.

Considering the linear Sharp model given by equations (**??**-**??**-**??**- **??**), the variation of the eigenvalues of the state matrix as a function of the longitudinal velocity v_x is given in figure (**??**). From the plots of eigenvalues (figure **??**), there are three distinct instability modes of the identified model, these modes of instability are differentiated by the speed at which they occur :

Wobble : This instability occurs mainly at high speed. It is between 9 and 15 Hz. This mode occurs when the handlebar begins to oscillate from one side to the other until the motorcycle falls.

Weave : It is an oscillating mode, it has a very low frequency which climbs to 4 Hz as the speed increases. It occurs between the rear wheel which leans and the front wheel which changes direction. It affects the whole motorcycle.

Capsize: Is a non-oscillating mode well damped at low speeds and with a decreasing damping in medium and high speeds. In this mode, the wheel is steered in the roll direction but not sufficiently to avoid the fall.

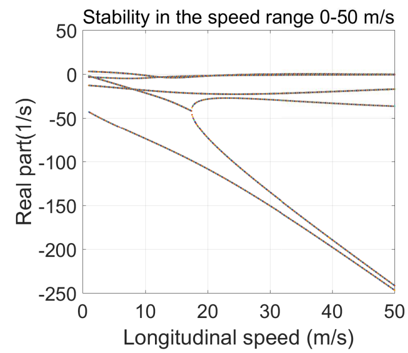

Fig. 6. The real parts of the eigenvalues of the identified Sharp model as a function of the longitudinal velocity.

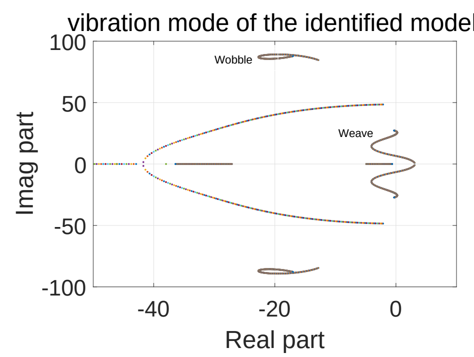

Fig. 7. The imaginary parts as a function of the real parts of the eigenvalues of the state matrix of the identified Sharp model.

5. Analyzing the Estimated Model

5..1. Theil Inequality Coefficient (TIC)

the Theil inequality coefficient (TIC) is the standardized root mean-squared error, used in the sensitivity analysis to measure the model predictive accuracy and to facilitate comparison between the actual and identified model.

The TIC is bounded by 0 and 1, the lower boundary is the ideal case of perfect forecast, zero being the case for which the model perfectly predicts the data, and 1 is the case for which the model has no predictive capability.

The following formula measure the forecast accuracy [?]:

$$TIC_i = \frac{\sqrt{\frac{1}{n} \cdot \sum (y_{i_m}(t_k) - y_i(t_k))^2}}{\sqrt{\frac{1}{n} \cdot \sum (y_{i_m}(t_k))^2} + \sqrt{\frac{1}{n} \cdot \sum (y_i(t_k))^2}} \quad (14)$$

where y_{i_m} are the actual output observations containing n samples and y_i are the corresponding predictions, resulting from estimated parameters.

TABLE I

THEIL INEQUALITY COEFFICIENT (TIC)

Output y_i	ϕ	δ	v_y	$\dot{\psi}$
TIC_i	0.0010	0.0016	0.0020	0.0012
Output y_i	$\dot{\phi}$	$\dot{\delta}$	F_{yf}	F_{yr}
TIC_i	0.0041	0.0158	0.0014	0.0015

Model output sensitivities are quantified in terms of output variation percentage, this coefficient is denoted "Fit", to compare the performance of the models that we have estimated.

The "Fit" value is calculated as follow:

$$Fit_i = 100. \frac{(1 - \|(y_i - y_{i_m})\|)}{\|(y_{i_m} - mean(y_{i_m}))\|} \quad (15)$$

TABLE II

FIT VALUES

Output y_i	ϕ	δ	v_y	$\dot{\psi}$
Fit_i %	99.7440	99.6054	99.5089	99.6370
Output y_i	$\dot{\phi}$	$\dot{\delta}$	F_{yf}	F_{yr}
Fit_i %	99.1790	96.8506	99.6540	99.6259

Good motorcycle models are vital for control concepts, and predicting risks to ensure safety for riders. From Tables 1 and 2, we can draw the following conclusions :
The small values of TIC, show a good forecast accuracy and prove the reliability of the model. The table of the fit value above indicate that the estimated parameters ensure a good model predictive characteristics. The accuracy influence is clearly checking.

6. CONCLUSION

A methodology for iterative, cascade identification of PTWv model has been proposed for the estimation of unknown parameters. Equations that describe the lateral dynamic are used to form the resulting problem that is solved using a multiple-objective optimization algorithm adapted to the complexity of our model.
In order to find the lateral dynamics we had to solve a linear gray-box problem by gradient method. The method has been successfully evaluated by simulation.

In future works, it is interesting to couple the identification and observer which may include additional parameters and state. Another perspective is to take into account the geometry of the road that has been considered flat and to consider the effect of the rider in the dynamics.

APPENDIX

The parameters defined in the table are similar to ones given in [?] and are listed in table **??**.

TABLE III

MOTORCYCLE DYNAMIC VARIABLES

Motorcycle	
τ	steering torque
M_f , M_r , M	mass of the front frame, the rear frame and the whole motorcycle
K	damper coefficient of the steering
Z_f , Z_r	front and rear vertical forces
C_{f1}, C_{r1}	front and rear tire cornering stiffness
C_{f2}, C_{r2}	front and rear tire camber stiffness
σ_f, σ_r	coefficients of relaxation of the front and rear pneumatic forces
j, h, k, e, l_f, l_r	linear dimensions
i_{fy}, i_{ry}	polar moment of inertia of front and rear wheels
R_f, R_r	radius of front and rear wheels
ε	caster angle
K	damper coefficient of the steering mechanism
η	mechanical trail
I_{fx}, I_{rx}	front and rear frame inertias about X axis
I_{fz}, I_{rz}	front and rear frame inertias about Z axis
C_{rxz}	rear frame product of inertia, X and Z axis
g	acceleration due to gravity

TABLE IV

MOTORCYCLE PARAMETERS EXPRESSIONS AND NUMERICAL VALUES

parameters θ_j contain the geometrical parameters of the motorcycle
$\theta_1 = M_f k$, $\theta_2 = M_f e$
$\theta_3 = M_f k^2 + I_{rz} + I_{fx} \sin^2 \varepsilon + I_{fz} \cos^2 \varepsilon$,
$\theta_4 = M_f jk - C_{rxz} + (I_{fz} - I_{fx}) \sin\varepsilon \cos\varepsilon$
$\theta_5 = M_f ek + I_{fz} \cos\varepsilon$, $\theta_6 = \frac{i_{fy}}{R_f} + \frac{i_{ry}}{R_r}$, $\theta_7 = \frac{i_{fy}}{R_f} \sin\varepsilon$ 1
$\theta_8 = M_f j^2 + M_r h^2 + I_{rx} + I_{fx} \cos^2 \varepsilon + I_{fz} \sin^2 \varepsilon$, $\theta_9 = M_f ej + I_{fz} \sin\varepsilon$
$\theta_{10} = -(M_f j + M_r h + \frac{i_{fy}}{R_f} + \frac{i_{ry}}{R_r})$, $\theta_{11} = -\frac{i_{fy}}{R_f} \cos\varepsilon$, $\theta_{12} = M_f eg - \eta Z_f$
$\theta_{13} = I_{fz} + M_f e^2$, $\theta_{14} = -(M_f e + \frac{i_{fy}}{R_f} \sin\varepsilon)$, $\theta_{15} = -K$

Numerical values of θ_j and physical parameters of the motorcycle deduced from the vector θ
$\theta_1 = 14.6685$, $\theta_2 = 0.1269$, $\theta_3 = 24.7957$, $\theta_4 = 5.0585$, $\theta_5 = 0.3441$, $\theta_6 = 4.3007$, $\theta_7 = 0.7774$, $\theta_8 = 68.0543$, $\theta_9 = 0.1310$, $\theta_{10} = -96.6900$, $\theta_{11} = -1.8126$, $\theta_{12} = 38.0886$, $\theta_{13} = 0.2010$, $\theta_{14} = -0.9721$, $\theta_{15} = -11.7332$ 1
$l = 1.3m$, $M_f = 16kg$, $M_r = 170kg$, $M = M_f + M_r$ $K = 11.7332 N.s/rad$, $g = 9.81 m/s^2$ $I_{fz} = 0.200 kg/m^2$, $i_{fy} = 0.4000 kg/m^2$, $i_{ry} = 0.4608 kg/m^2$ $a = 0.949m$, $e = 0.0079m$, $f = -0.1527m$, $h = 0.509m$, $R_f = 0.2m$, $R_r = 0.2m$, $\eta = 0.08m$, $\varepsilon = 0.4363 rad$ $k = 0.9168$, $j = 0.2589$ $l_r = 0.35m$, $l_f = 0.95m$, $emp = l_f + l_r$, $Z_f = -491.25$, $C_{f1} = 18592$, $C_{f2} = 1195.2$, $C_{r1} = 19209.58$, $C_{r2} = 960.48$ $\sigma_f = 0.2$, $\sigma_r = 0.2$

The system equation used in the identification algorithm :

$$d_v = \left(\dot{v}_y + v_x \dot{\psi}\right), \; d_x = \sin(\phi) + \sin(\varepsilon)\sin(\delta)$$

$$
\begin{cases}
y_1 = -\frac{(Mh\ddot{\phi}+Mv_x\dot{\psi}+M\dot{v}_y-Ma_y)}{\theta_1} \\
y_2 = -\frac{(Mh\ddot{\phi}+\theta_1\ddot{\psi}+Mv_x\dot{\psi}+M\dot{v}_y-Ma_y)}{\theta_2} \\
y_3 = \frac{-\theta_1 d_v-\theta_1 v_x\dot{\psi}+l_f F_{yf}-l_r F_{yr}}{\theta_3} \\
y_4 = \frac{-\theta_1 d_v-\theta_3\ddot{\psi}-\theta_1 v_x\dot{\psi}+l_f F_{yf}-l_r F_{yr}}{\theta_4} \\
y_5 = \frac{-\theta_1 d_v-\theta_3\ddot{\psi}-\theta_4\ddot{\phi}+l_f F_{yf}-l_r F_{yr}}{\theta_5} \\
y_6 = \frac{-\theta_1 d_v-\theta_3\ddot{\psi}-\theta_4\ddot{\phi}-\theta_5\ddot{\delta}+l_f F_{yf}-l_r F_{yr}}{v_x\theta_6} \\
y_7 = \frac{-\theta_1 d_v-\theta_3\ddot{\psi}-\theta_4\ddot{\phi}-\theta_5\ddot{\delta}+\theta_6 v_x\dot{\phi}+l_f F_{yf}-l_r F_{yr}}{v_x\theta_7} \\
y_8 = \frac{-Mh\dot{v}_y+\theta_4\ddot{\psi}+\theta_9\ddot{\delta}-\theta_{10}v_x\dot{\psi}-\theta_{11}v_x\dot{\delta}+Mhg\sin(\phi)+\theta_{12}\sin(\delta))}{\theta_8} \\
y_9 = \frac{-(\theta_2\dot{v}_y+\theta_9\ddot{\phi}+\theta_5\ddot{\psi}+\theta_{13}\ddot{\delta}+\theta_{11}v_x\dot{\phi}-\theta_{14}v_x\dot{\psi}+\theta_{12}d_x-\eta F_{yf}+\tau)}{\theta_{15}}
\end{cases}
\tag{16}
$$

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